PROBLEMS

10-1. Calculate the centrifugal acceleration, due to Earth's rotation, on a particle on the surface of Earth at the equator. Compare this result with the gravitational acceleration. Compute also the centrifugal acceleration due to the motion of Earth about the Sun and justify the remark made in the text that this acceleration may be neglected compared with the acceleration caused by axial rotation.

10-2. An automobile drag racer drives a car with acceleration a and instantaneous velocity v. The tires (of radius r_0) are not slipping. Find which point on the tire has the greatest acceleration relative to the ground. What is this acceleration?

10-3. In Example 10.2, assume that the coefficient of static friction between the hockey puck and a horizontal rough surface (on the merry-go-round) is μ_s. How far away from the center of the merry-go-round can the hockey puck be placed without sliding?

10-4. In Example 10.2, for what initial velocity and direction in the rotating system will the hockey puck appear to be subsequently motionless in the fixed system? What will be the motion in the rotating system? Let the initial position be the same as in Example 10.2. You may choose to do a numerical calculation.

10-5. Perform a numerical calculation using the parameters in Example 10.2 and Figure 10-4e, but find the initial velocity for which the path of motion passes back over the initial position in the rotating system. At what time does the puck exit the merry-go-round?

10-6. A bucket of water is set spinning about its symmetry axis. Determine the shape of the water in the bucket.

10-7. Determine how much greater the gravitational field strength g is at the pole than at the equator. Assume a spherical Earth. If the actual measured difference is $\Delta g = 52\ \text{mm/s}^2$, explain the difference. How might you calculate this difference between the measured result and your calculation?

10-8. If a particle is projected vertically upward to a height h above a point on Earth's surface at a northern latitude λ, show that it strikes the ground at a point $\frac{4}{3}\,\omega\cos\lambda\cdot\sqrt{8h^3/g}$ to the west. (Neglect air resistance, and consider only small vertical heights.)

10-9. If a projectile is fired due east from a point on the surface of Earth at a northern latitude λ with a velocity of magnitude V_0 and at an angle of inclination to the horizontal of α, show that the lateral deflection when the projectile strikes Earth is

$$d = \frac{4V_0^3}{g^2}\cdot\omega\sin\lambda\cdot\sin^2\alpha\cos\alpha$$

where ω is the rotation frequency of Earth.

10-10. In the preceding problem, if the range of the projectile is R_0' for the case $\omega = 0$, show that the change of range due to the rotation of Earth is

$$\Delta R' = \sqrt{\frac{2R_0'^3}{g}} \cdot \omega \cos \lambda \left(\cot^{1/2}\alpha - \frac{1}{3} \tan^{3/2}\alpha \right)$$

10-11. Obtain an expression for the angular deviation of a particle projected from the North Pole in a path that lies close to Earth. Is the deviation significant for a missile that makes a 4,800-km flight in 10 minutes? What is the "miss distance" if the missile is aimed directly at the target? Is the miss distance greater for a 19,300-km flight at the same velocity?

10-12. Show that the small angular deviation ε of a plumb line from the true vertical (i.e., toward the center of Earth) at a point on Earth's surface at a latitude λ is

$$\varepsilon = \frac{R\omega^2 \sin \lambda \cos \lambda}{g_0 - R\omega^2 \cos^2 \lambda}$$

where R is the radius of Earth. What is the value (in seconds of arc) of the maximum deviation? Note that the entire denominator in the answer is actually the effective g, and g_0 denotes the pure gravitational component.

10-13. Refer to Example 10.3 concerning the deflection from the plumb line of a particle falling in Earth's gravitational field. Take g to be defined at ground level and use the zeroth order result for the time-of-fall, $T = \sqrt{2h/g}$. Perform a calculation in second approximation (i.e., retain terms in ω^2) and calculate the *southerly* deflection. There are three components to consider: **(a)** Coriolis force to second order (C_1), **(b)** variation of centrifugal force with height (C_2), and **(c)** variation of gravitational force with height (C_3). Show that each of these components gives a result equal to

$$C_i \frac{h^2}{g} \omega^2 \sin \lambda \cos \lambda$$

with $C_1 = 2/3$, $C_2 = 5/6$, and $C_3 = 5/2$. The total southerly deflection is therefore $(4h^2\omega^2 \sin \lambda \cos \lambda)/g$.

10-14. Refer to Example 10.3 and the previous problem, but drop the particle at Earth's surface down a mineshaft to a depth h. Show that in this case there is no southerly deflection due to the variation of gravity and that the total southerly deflection is only

$$\frac{3}{2} \frac{h^2\omega^2}{g} \sin \lambda \cos \lambda$$

10-15. Consider a particle moving in a potential $U(r)$. Rewrite the Lagrangian in terms of a coordinate system in uniform rotation with respect to an inertial frame. Calculate the Hamiltonian and determine whether $H = E$. Is H a constant of the motion? If E is not a constant of motion, why isn't it? The expression for the Hamiltonian thus obtained is the standard formula $1/2\ mv^2 + U$ plus an additional term. Show that the extra term is the *centrifugal potential energy*. Use the Lagrangian you obtained to reproduce the equations of motion given in Equation 10.25 (without the second and third terms).

10-16. Consider Problem 9-63 but include the effects of the Coriolis force on the probe. The probe is launched at a latitude of 45° straight up. Determine the horizontal deflection in the probe at its maximum height for each part of Problem 9-63.

10-17. Approximate Lake Superior by a circle of radius 162 km at a latitude of 47°. Assume the water is at rest with respect to Earth and find the depth that the center is depressed with respect to the shore due to the centrifugal force.

10-18. A British warship fires a projectile due south near the Falkland Islands during World War I at latitude 50°S. If the shells are fired at 37° elevation with a speed of 800 m/s, by how much do the shells miss their target and in what direction? Ignore air resistance.

10-19. Find the Coriolis force on an automobile of mass 1300 kg driving north near Fairbanks, Alaska (latitude 65°N) at a speed of 100 km/h.

10-20. Calculate the effective gravitational field vector **g** at Earth's surface at the poles and the equator. Take account of the difference in the equatorial (6378 km) and polar (6357 km) radius as well as the centrifugal force. How well does the result agree with the difference calculated with the result $g = 9.780356[1 + 0.0052885 \sin^2\lambda - 0.0000059 \sin^2(2\lambda)]\,\text{m/s}^2$ where λ is the latitude?

10-21. Water being diverted during a flood in Helsinki, Finland (latitude 60°N) flows along a diversion channel of width 47 m in the south direction at a speed of 3.4 m/s. On which side is the water the highest (from the standpoint of noninertial systems) and by how much?

10-22. Shot towers were popular in the eighteenth and nineteenth centuries for dropping melted lead down tall towers to form spheres for bullets. The lead solidified while falling and often landed in water to cool the lead bullets. Many such shot towers were built in New York State. Assume a shot tower was constructed at latitude 42°N, and the lead fell a distance of 27 m. In what direction and how far did the lead bullets land from the direct vertical?