IF/UFRJ
Statistical Mechanics - 2025/1 – Raimundo

Problem Set #3 – The Canonical Ensemble – Part 1

7/4/2024

1. A system with $N(\gg 1)$ uncoupled oscillators and total energy E is in thermal equilibrium. Using the result of Prob. 1 of Set #2, obtain the probability that *one* oscillator is in state n, with energy $\varepsilon_n = (n + 1/2)h\nu$. Comment on your findings.

2. (a) Derive the following Maxwell relation for a fluid system:
$$\left(\frac{\partial \mu}{\partial P}\right)_{T,N} = \left(\frac{\partial V}{\partial N}\right)_{T,P} = v,$$
where $v \equiv V/N = 1/n$.

   (b) Show that for an ideal gas at a given temperature T, the chemical potential difference between an arbitrary pressure, P, and a reference pressure, P_0, is given by
$$\mu(P) - \mu(P_0) = k_B T \ln\left(P/P_0\right).$$

   (c) For the case of an incompressible liquid, the volume per particle, $v \equiv V/N$, is independent of the pressure; show that in this case, one has
$$\mu(P) - \mu(P_0) = v\,(P - P_0).$$

   (d) Discuss the features common to the results obtained in (b) and (c).

3. On every one of the N $(\gg 1)$ sites of a linear chain there is a spin-1/2. The interaction energy of any pair of nearest neighbour spins (located at sites i and $i + 1$) may be written as
$$\varepsilon_{i,i+1} = -J\sigma_i\sigma_{i+1}, \tag{1}$$
where J is a constant with dimension of energy, and $\sigma_j = \pm 1$ is a measure of the orientation of the magnetic moment. Therefore, pairs of parallel (antiparallel)

spins contribute with $-J\,(+J)$ to the total energy. Admit that the system is initially in the ground state ($T = 0$), so that all spins are parallel (either all 'up' or all 'down')

(a) Which are the energy (E_0), the entropy (S_0), and the Helmholtz free energy associated with this configuration?

(b) Consider now a possible lowest energy excited state, such that all spins to the right of any site have been flipped (see Fig. 1). Determine the energy, $E\,'$, and entropy, $S\,'$ associated with this new situation, as if the temperature were zero.

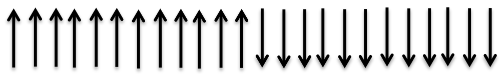

Figure 1: Problem 3

(c) We admit that this new configuration appeared as a result of thermally induced fluctuations. At low temperatures, the Helmholtz free energy associated with this new configuration may be given by

$$A' = E' - TS'. \tag{2}$$

What is the change in free energy, $\Delta A = A' - A$?

(d) What can one conclude about the influence of temperature on the stability of the configuration with parallel spins?