IF/UFRJ
Statistical Mechanics - 2024/1 – Raimundo

Problem Set #3 – The Canonical Ensemble – Part 1

25/3/2024

1. A system with $N(\gg 1)$ uncoupled oscillators and total energy E is in thermal equilibrium. Using the result of Prob. 1 of Set #2, obtain the probability that *one* oscillator is in state n, with energy $\varepsilon_n = (n+1/2)h\nu$. Comment on your findings.

2. (a) Derive the following Maxwell relation for a fluid system:

$$\left(\frac{\partial \mu}{\partial P}\right)_{T,N} = \left(\frac{\partial V}{\partial N}\right)_{T,P} = v,$$

   where $v \equiv V/N = 1/n$.

   (b) Show that for an ideal gas at a given temperature T, the chemical potential difference between an arbitrary pressure, P, and a reference pressure, P_0, is given by

$$\mu(P) - \mu(P_0) = k_B T \ln\left(P/P_0\right).$$

   (c) For the case of an incompressible liquid, the volume per particle, $v \equiv V/N$, is independent of the pressure; show that in this case, one has

$$\mu(P) - \mu(P_0) = v\,(P - P_0).$$

   (d) Discuss the features common to the results obtained in (b) and (c).

3. On every one of the N ($\gg 1$) sites of a linear chain there is a spin-1/2. The interaction energy of any pair of nearest neighbour spins (located at sites i and $i+1$) may be written as

$$\varepsilon_{i,i+1} = -J\sigma_i\sigma_{i+1}, \tag{1}$$

   where J is a constant with dimension of energy, and $\sigma_j = \pm 1$ is a measure of the orientation of the magnetic moment. Therefore, pairs of parallel (antiparallel) spins contribute with $-J\,(+J)$ to the total energy. Admit that the system is initially in the ground state ($T = 0$), so that all spins are parallel (either all 'up' or all 'down')

(a) Which are the energy (E_0), the entropy (S_0), and the Helmholtz free energy associated with this configuration?

(b) Consider now a possible lowest energy excited state, such that all spins to the right of any site have been flipped (see Fig. 1). Determine the energy, E', and entropy, S' associated with this new situation, as if the temperature were zero.

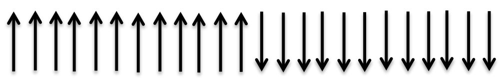

Figure 1: Problem 3

(c) We admit that this new configuration appeared as a result of thermally induced fluctuations. At low temperatures, the Helmholtz free energy associated with this new configuration may be given by

$$A' = E' - TS'. \tag{2}$$

What is the change in free energy, $\Delta A = A' - A$?

(d) What can one conclude about the influence of temperature on the stability of the configuration with parallel spins?