# Quantum decoherence, Casimir effect, quantum vacuum fluctuations

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École Doctorale QMat 182 - U. Strasbourg

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# Quantum decoherence, Casimir effect, quantum vacuum fluctuations

11/12/2023

Lecture 1: Decoherence of massive particles by radiation pressure: introduction

20/12/2023

Lecture 2

Part A (cont of Lecture 1):

Casimir effect, decoherence via master equation

Part B:

Dynamical Casimir effects with atoms

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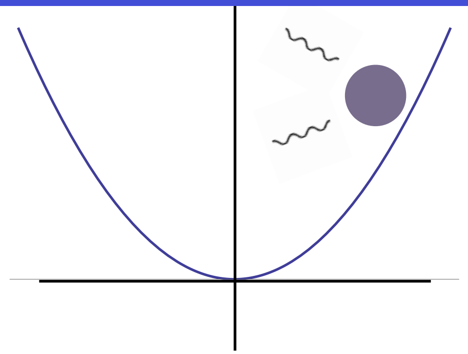
# Decoherence by radiation pressure: master equation, results at zero and finite temperatures

# decoherence by radiation pressure

Master equation for the particle center of mass:



- radiation pressure coupling: quadratic in the electromagnetic field operators
- external harmonic potential (optical tweezer): frequency Ω

⇨ master equation for reduced density operator of the bead CM

Similar to quantum Brownian motion models -

Caldeira & Leggett (1985), Unruh & Zurek (1989), Hu, Paz & Zhang (1992)

Thermal field radiation pressure coupling: Joos & Zeh (1985)

$$\partial_t \hat{\rho}(t) = \boxed{-\frac{i}{\hbar}[\hat{H}_0, \hat{\rho}(t)]} \boxed{-\frac{i}{\hbar}\gamma(t)\,\underbrace{[\hat{x}, \{\hat{p}, \hat{\rho}(t)\}]}_{\text{damping}} - \frac{1}{\hbar^2} D(t)\,\underbrace{[\hat{x}, [\hat{x}, \hat{\rho}(t)]]}_{\text{diffusion}}}$$

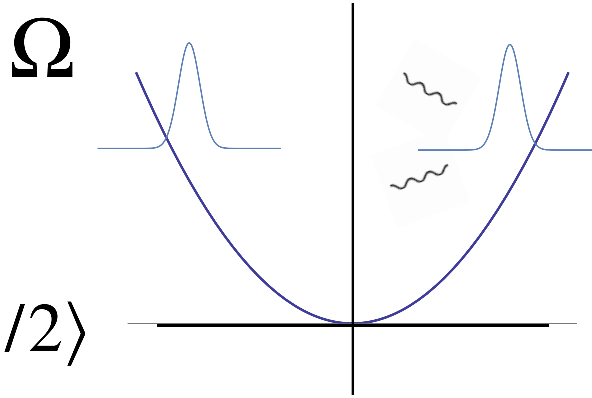
unitary evolution

[environment = quantum electromagnetic field]

DAR Dalvit and PAMN, Phys. Rev. Lett. **84** 799 (2000); Phys. Rev. A**62**, 042103 (2000)

# decoherence by radiation pressure

Master equation for the particle center of mass: equivalent to Fokker-Planck equation for the Wigner function $W(x, p, t)$



$$W(x, p, t) \equiv \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} dy\, e^{ipy/\hbar} \langle x - y/2 | \hat{\rho} | x + y/2 \rangle$$

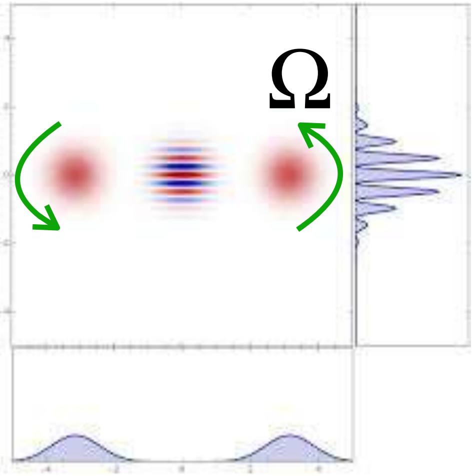
$$\partial_t W = \boxed{-\frac{p}{m}\partial_x W + m\Omega^2 x \partial_p W} + \underbrace{\gamma \partial_p (pW)}_{\text{damping}} + \underbrace{D \frac{\partial^2 W}{\partial p^2}}_{\text{diffusion}}$$

Unitary rotation in phase space

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}} \left( |\alpha_0\rangle + |-\alpha_0\rangle \right)$$

$$|\psi\rangle_t = \frac{1}{\sqrt{2}} \left( |\alpha(t)\rangle + |-\alpha(t)\rangle \right)$$

$$\alpha(t) = \alpha_0\, e^{-i\Omega t}$$



DAR Dalvit and PAMN, Phys. Rev. Lett. **84** 799 (2000); Phys. Rev. A**62**, 042103 (2000)

# decoherence by radiation pressure

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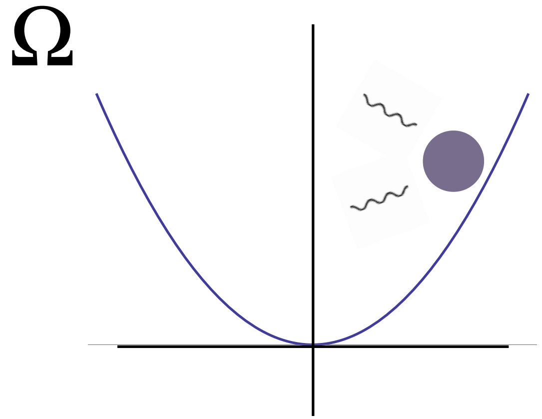
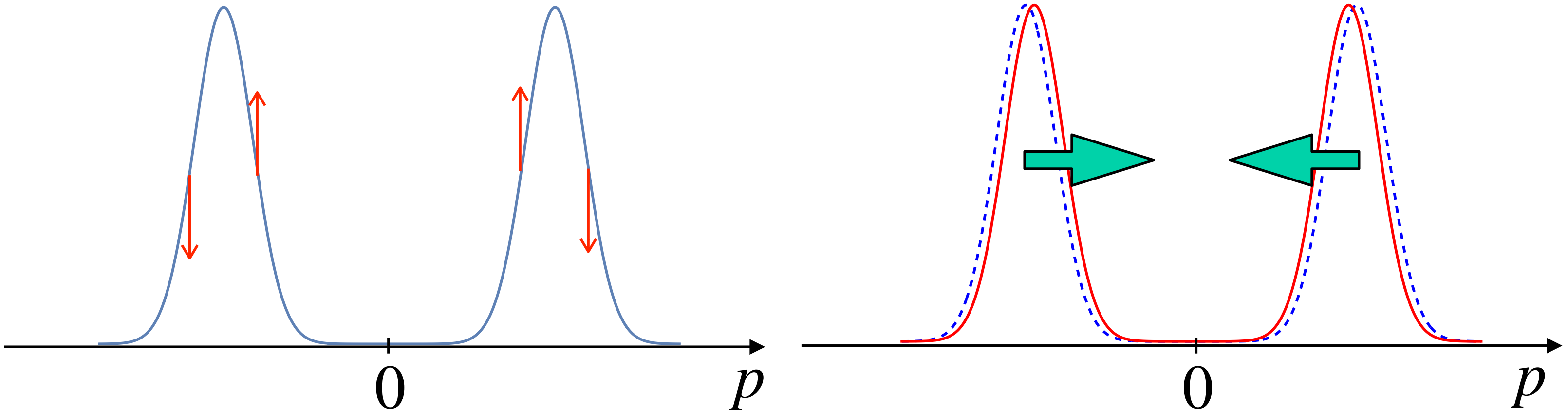


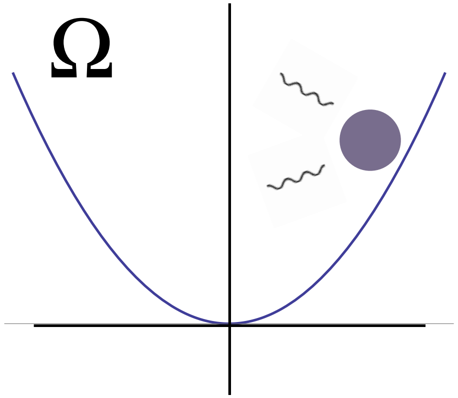
$$\partial_t W = -\frac{p}{m}\partial_x W + m\Omega^2 x\partial_p W + \boxed{\gamma\partial_p(pW)} + D\frac{\partial^2 W}{\partial p^2}$$

**damping**



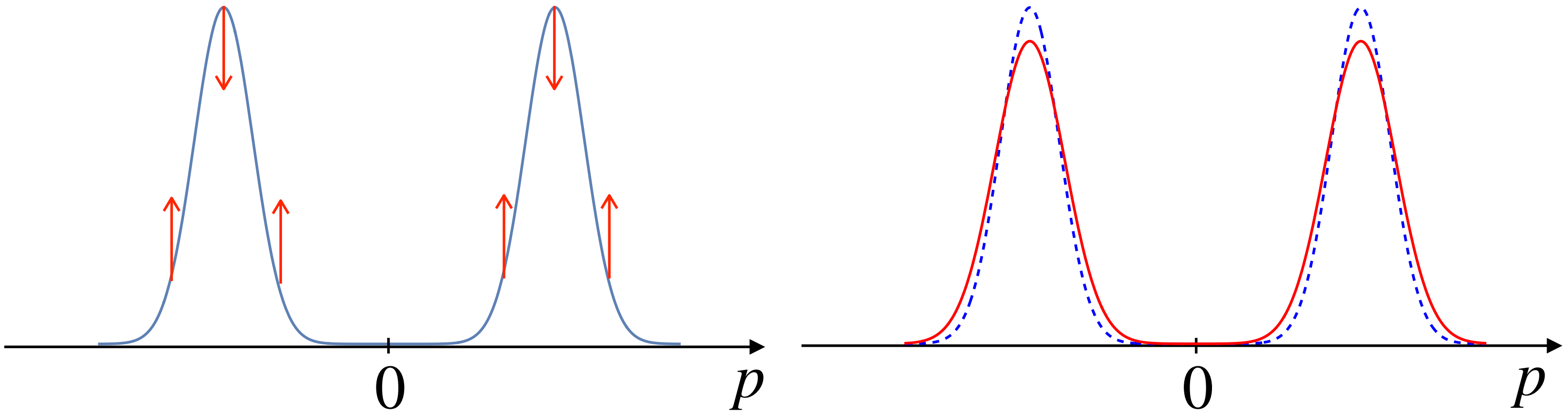
# decoherence by radiation pressure

Master equation for the particle center of mass: equivalent to Fokker-Planck equation for the Wigner function $W(x, p, t)$



$$\partial_t W = -\frac{p}{m}\partial_x W + m\Omega^2 x \partial_p W + \gamma\partial_p(pW) + \boxed{D\frac{\partial^2 W}{\partial p^2}}$$

**diffusion**



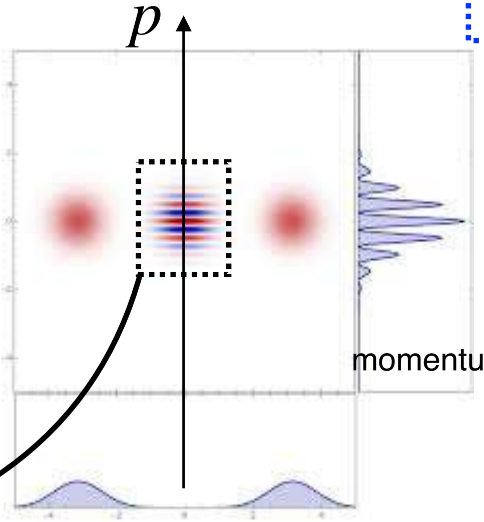
# decoherence by radiation pressure

Master equation for the particle center of mass: equivalent to Fokker-Planck equation for the Wigner function $W(x, p, t)$

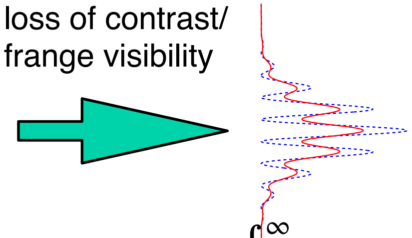
$$\partial_t W = -\frac{p}{m}\partial_x W + m\Omega^2 x\partial_p W + \gamma\partial_p(pW) + \boxed{D\frac{\partial^2 W}{\partial p^2}}$$

Initial state:
superposition of
coherent states

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}\big(|\alpha_0\rangle + |-\alpha_0\rangle\big)$$

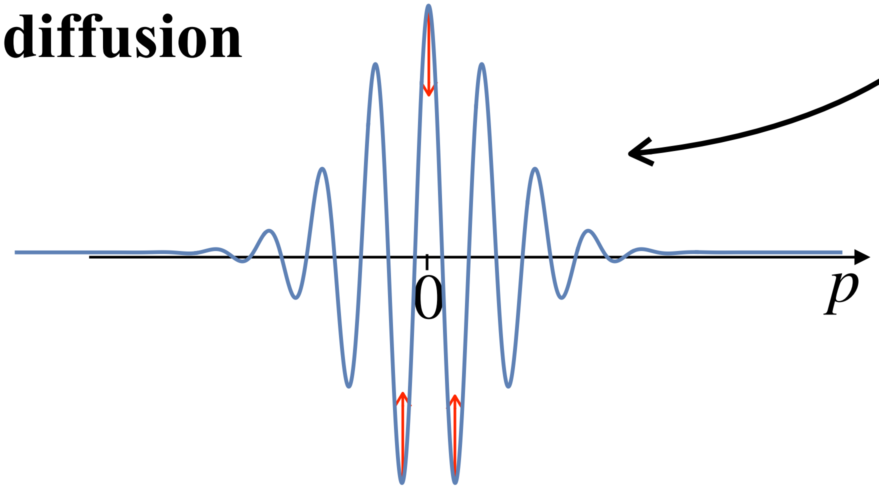


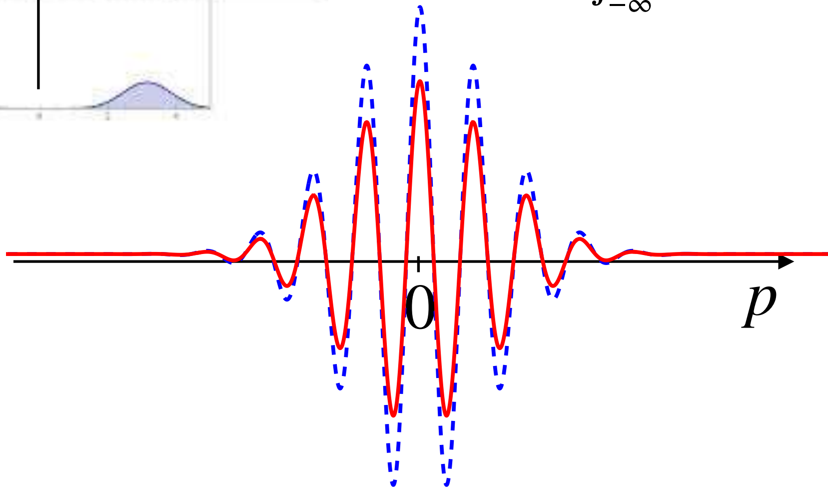
loss of contrast/
frange visibility



momentum prob density = $\int_{-\infty}^{\infty} dx\, W(x,p)$

**diffusion**

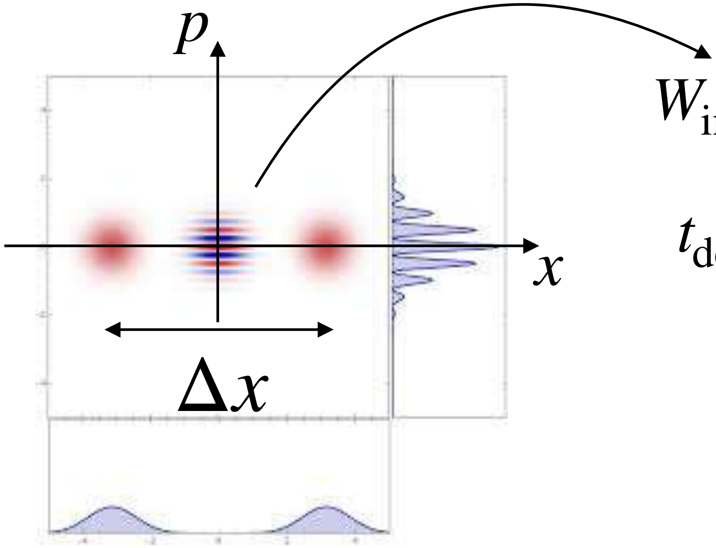




# decoherence by radiation pressure

Decoherence from diffusion in phase space

$$\partial_t W = -\frac{p}{m}\partial_x W + m\Omega^2 x \partial_p W + \gamma \partial_p (pW) + \boxed{D\frac{\partial^2 W}{\partial p^2}}$$



$$W_{\rm int}(x,p,t) \sim \cos(\Delta x\, p/\hbar)\ \exp(-t/t_{\rm dec})$$

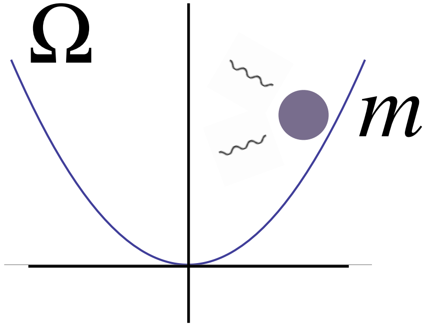
$t_{\rm dec}$ is the decoherence time

$$\frac{1}{t_{\rm dec}} = D\left(\frac{\Delta x}{\hbar}\right)^2$$

The less classical the state is, the faster is decoherence

# decoherence by radiation pressure

## Damping and diffusion coefficients



Weak coupoling to the environment

Long times $t \gg 2\pi/\Omega$: damping and diffusion coefficients become constant

Anti-symmetric correlation function

$$\xi(t) \equiv \langle [F(t), F(0)] \rangle \Longrightarrow \gamma \approx \frac{1}{4m\hbar\Omega}\tilde{\xi}(\Omega)$$

Symmetric correlation function

$$\sigma(t) \equiv \langle \{F(t), F(0)\} \rangle \Longrightarrow D \approx \frac{1}{4}\tilde{\sigma}(\Omega)$$

Fluctuation-dissipation theorem when **environment** is at thermal equilibrium, temperature $T$

$$\tilde{\sigma}(\Omega) = \coth\left(\frac{\hbar\Omega}{2k_BT}\right)\tilde{\xi}(\Omega)$$

Diffusion vs damping:

High temperature $k_BT \gg \hbar\Omega$

$$D = 2k_BT\, m \boxed{\gamma(T)}$$

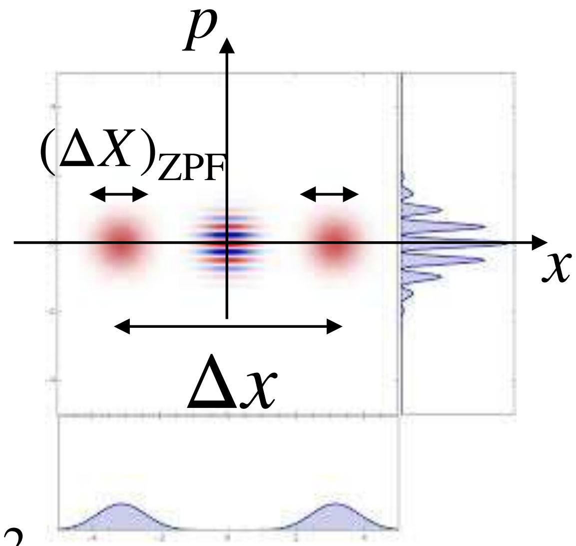
radiation pressure damping depends on temperature!

Low temperature $k_BT \ll \hbar\Omega$

$$D = \hbar\Omega\, m\, \gamma(T = 0)$$

DAR Dalvit and PAMN, Phys. Rev. Lett. **84** 799 (2000); Phys. Rev. A**62**, 042103 (2000)

# decoherence by radiation pressure



Decoherence time $t_{\mathrm{dec}}$

$$\frac{1}{t_{\mathrm{dec}}} = D\left(\frac{\Delta x}{\hbar}\right)^2$$

Low temperature $k_B T \ll \hbar\Omega$

Position uncertainty of ground state $(\Delta X)_{\mathrm{ZPF}}$:

$$(\Delta X)_{\mathrm{ZPF}} = \sqrt{\frac{\hbar}{2m\Omega}} \qquad \frac{1}{t_{\mathrm{dec}}} = \left(\frac{\Delta x}{(\Delta X)_{\mathrm{ZPF}}}\right)^2 \gamma(T=0)$$

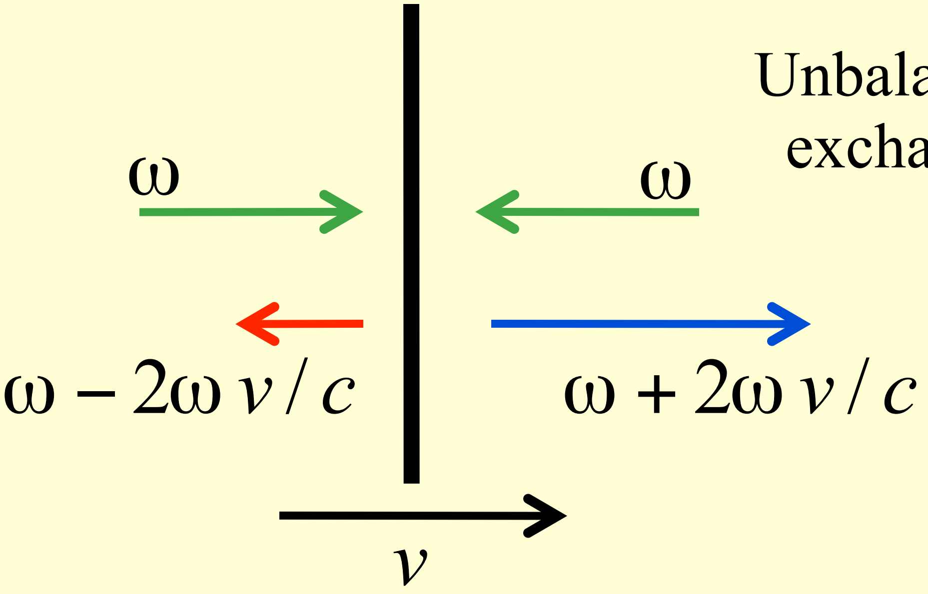
High temperature $k_B T \gg \hbar\Omega$: effect of thermal 'black-body' photons

Thermal de Broglie wavelength $\lambda_T = \dfrac{\hbar}{\sqrt{2mk_B T}}$

$$\frac{1}{t_{\mathrm{dec}}} = \left(\frac{\Delta x}{\lambda_T}\right)^2 \gamma(T) = \frac{k_B T}{\hbar\Omega}\left(\frac{\Delta x}{(\Delta X)_{\mathrm{ZPF}}}\right)^2 \gamma(T)$$

# Physical origin of the thermal drag force: Doppler effect

Take uniform velocity $v$



Unbalanced momentum exchange per photon:

$$\boxed{\Delta P = \frac{\hbar}{c} 2\omega \frac{v}{c}}$$

Recoil force

$$\boxed{\Rightarrow F = -\frac{\Delta P}{\Delta t} = -2\frac{\Delta E}{\Delta t}\frac{v}{c^2}}$$

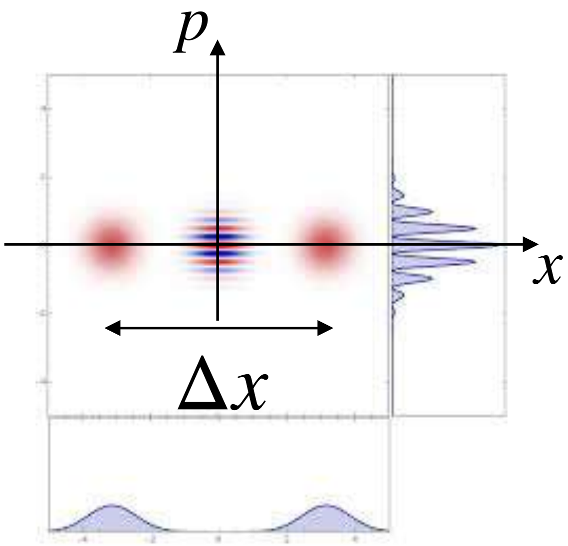
Using 3D density of modes and thermal photon number…

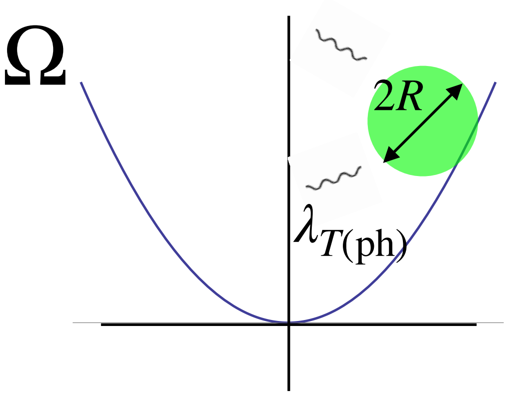
$$\boxed{F_T = -\frac{2\pi^2}{15} A \frac{(k_B T)^4}{\hbar^3 c^4} v}$$

# decoherence by radiation pressure

High temperature

$k_B T \gg \hbar\Omega$





We consider large spheres: semiclassical Mie scattering regime of black-body photons

$$R \gg \lambda_{T(\mathrm{ph})} = \frac{\hbar c}{k_B T} \approx 7.6\,\mu\mathrm{m} \quad @\, T = 300\,\mathrm{K}$$

$$\frac{1}{t_{\mathrm{dec}}} = \frac{8\pi^3}{45} \frac{c\, R^2 (\Delta x)^2}{\left(\lambda_{T(\mathrm{ph})}\right)^5} \Longrightarrow t_{\mathrm{dec}} = 0.15\,\mu\mathrm{s} \quad @\, R = 10\,\mu\mathrm{m}, \Delta x = 1\,\mathrm{nm}$$

$$t_{\mathrm{dec}} = 36\,\mu\mathrm{s} \quad @\, T = 100\,\mathrm{K}$$

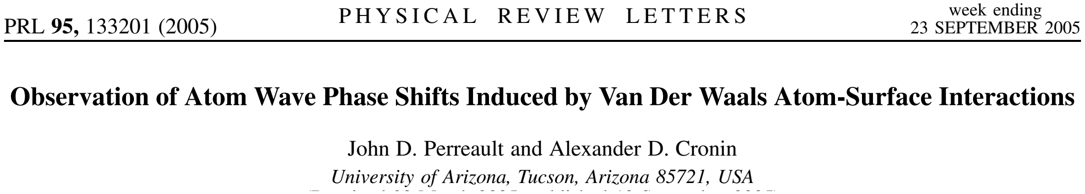
## Part B) Dynamical Casimir effects with atoms

- Geometric and non-local Casimir atomic phases
- Quantum Sagnac Effect

## Conclusion

# Geometric and non-local Casimir atomic phases

# introduction: atom interferometers



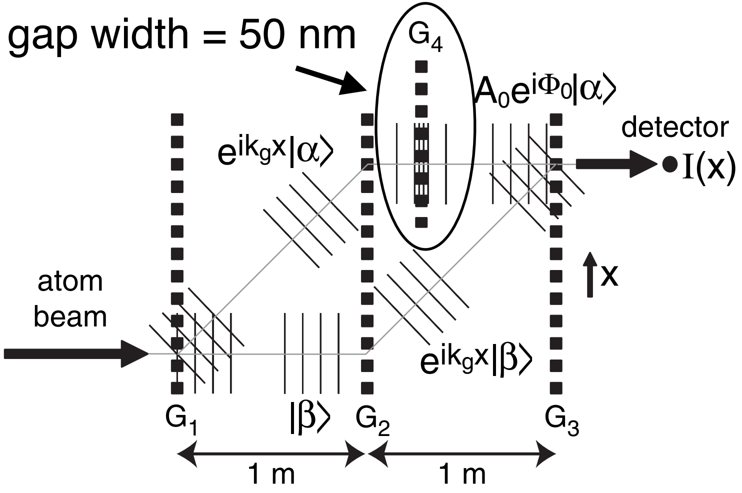
PRL **95,** 133201 (2005) PHYSICAL REVIEW LETTERS week ending 23 SEPTEMBER 2005

### Observation of Atom Wave Phase Shifts Induced by Van Der Waals Atom-Surface Interactions

John D. Perreault and Alexander D. Cronin
*University of Arizona, Tucson, Arizona 85721, USA*

Atom-Surface interaction in the nano grating

Shift of the atomic fringes



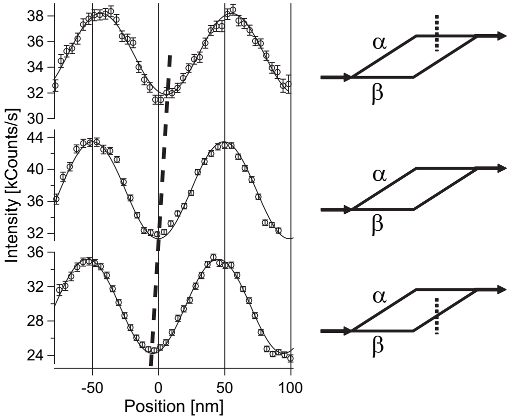
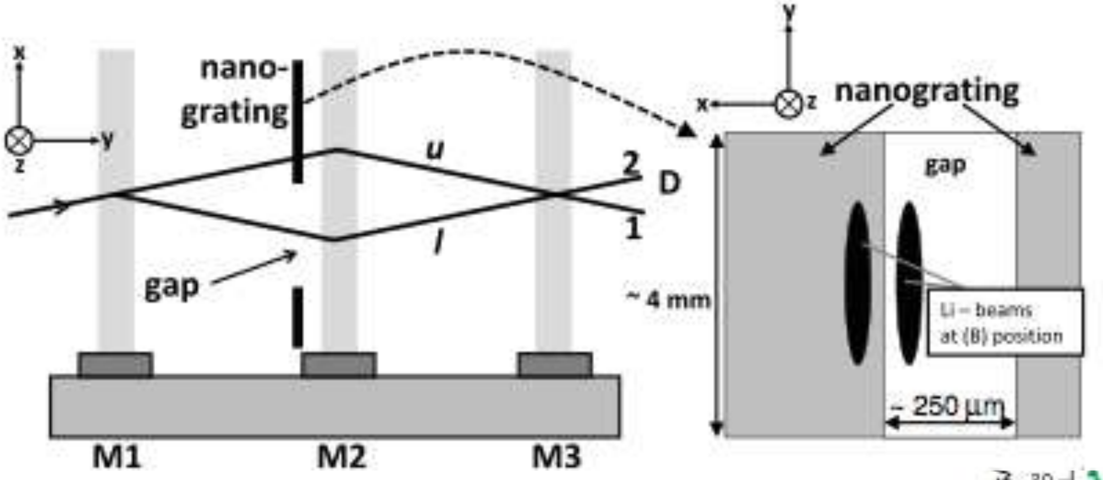


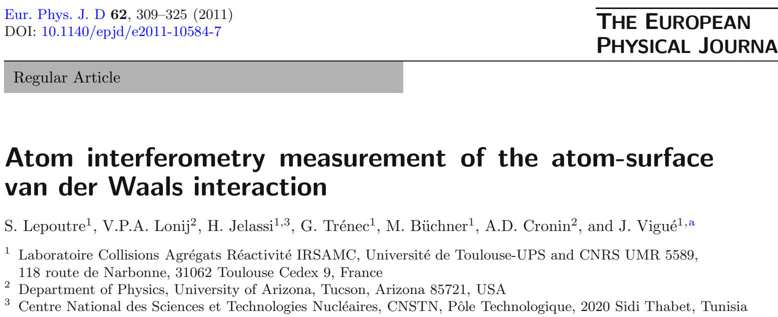
FIG. 3. Interference pattern observed when the grating $G_4$ is inserted into path $\alpha$ or $\beta$ of the atom interferometer. Each interference pattern represents 5 s of data. The intensity error bars are arrived at by assuming Poisson statistics for the number of detected atoms. The dashed line in the plots is a visual aid to help illustrate the measured phase shift of 0.3 rad. Notice how the phase shift induced by placing $G_4$ in path $\alpha$ or $\beta$ has opposite sign. The sign of the phase shift is also consistent with the atom experiencing an attractive potential as it passes through $G_4$.

# introduction: atom interferometers

## Bragg atom interferometer



John D. Perreault and Alexander D. Cronin, PRL **95**, 133201 (2005)
S. Lepoutre, H. Jelassi, V.P.A. Lonig, G. Trénec, M. Büchner, A. D. Cronin, and J. Vigué, EPL **88**, 20002 (2009)
S. Lepoutre et al. , EPJD **62**, 309 (2011)


Eur. Phys. J. D **62**, 309–325 (2011)
DOI: 10.1140/epjd/e2011-10584-7

THE EUROPEAN
PHYSICAL JOURNA

Regular Article

**Atom interferometry measurement of the atom-surface van der Waals interaction**

S. Lepoutre[1], V.P.A. Lonij[2], H. Jelassi[1,3], G. Trénec[1], M. Büchner[1], A.D. Cronin[2], and J. Vigué[1,a]

[1] Laboratoire Collisions Agrégats Réactivité IRSAMC, Université de Toulouse-UPS and CNRS UMR 5589, 118 route de Narbonne, 31062 Toulouse Cedex 9, France
[2] Department of Physics, University of Arizona, Tucson, Arizona 85721, USA
[3] Centre National des Sciences et Technologies Nucléaires, CNSTN, Pôle Technologique, 2020 Sidi Thabet, Tunisia

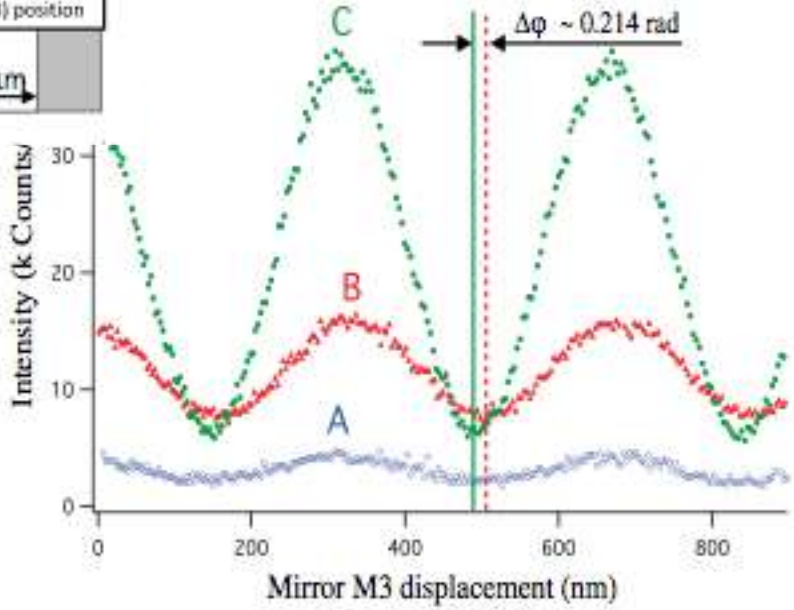
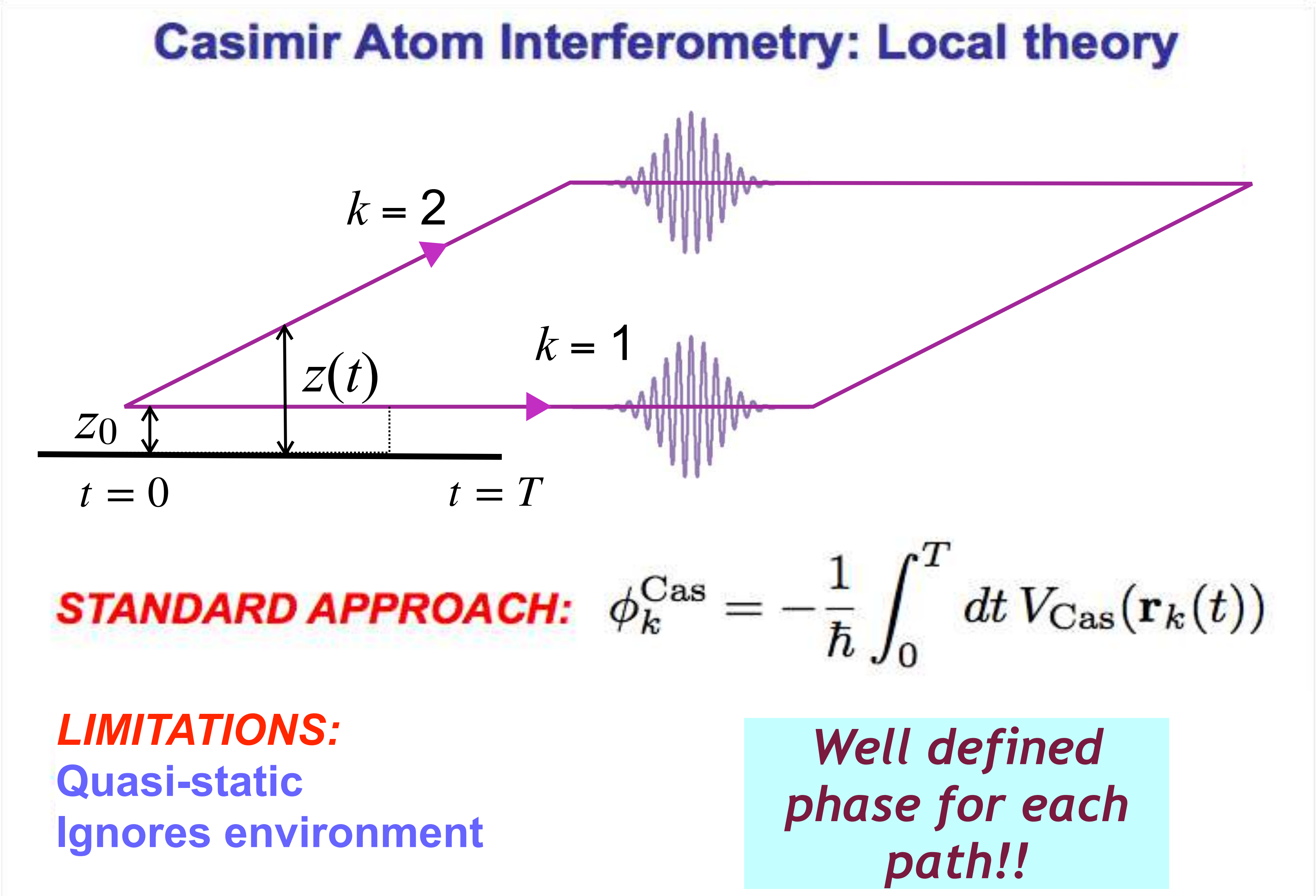


Fig. 2: (Colour on-line) Atom interference fringes recorded with (A) both arms (visibility $\mathcal{V}_A = 32\%$), (B) one arm ($\mathcal{V}_B = 34\%$), or (C) neither arm ($\mathcal{V}_C = 72\%$) passing through the nanostructure, with a lithium beam velocity $v = 1062 \pm 20\,\mathrm{m/s}$. The counting period is 0.1 s per data point.

# introduction: atom interferometers



## Casimir Atom Interferometry: Local theory

**STANDARD APPROACH:** $\phi_k^{\mathrm{Cas}} = -\frac{1}{\hbar}\int_0^T dt\, V_{\mathrm{Cas}}(\mathbf{r}_k(t))$

***LIMITATIONS:***

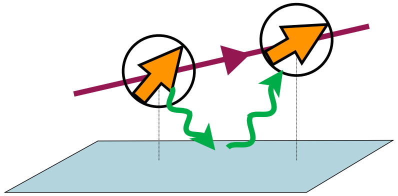
**Quasi-static**

**Ignores environment**

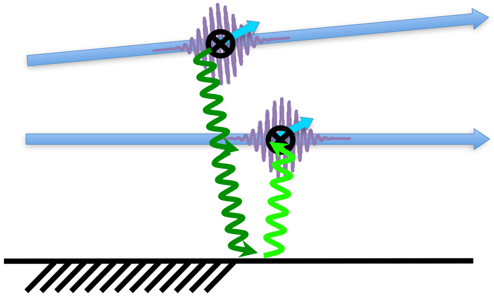
***Well defined phase for each path!!***

# non-local Casimir phase

atom-surface van der Waals interaction:
fluctuating dipole interacts with its **own field**, after reflection by surface



**interferometer: self-interaction also with a different wave-packet component**



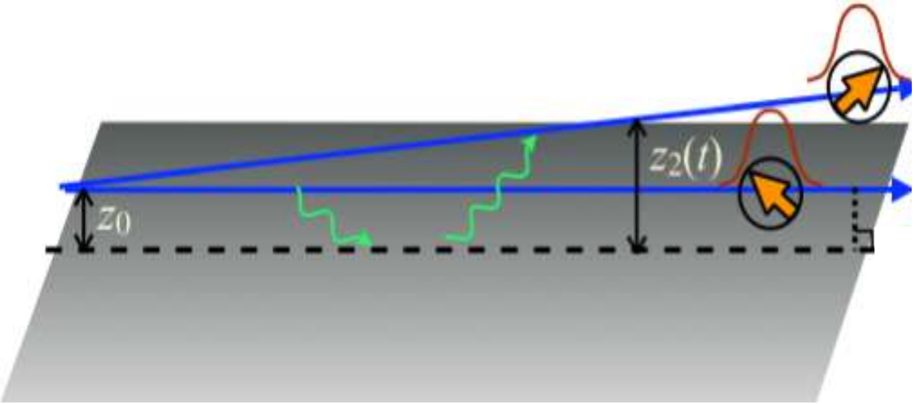
F Impens, R Behunin, C Ccapa-Ttira and PAMN, EPL 2013

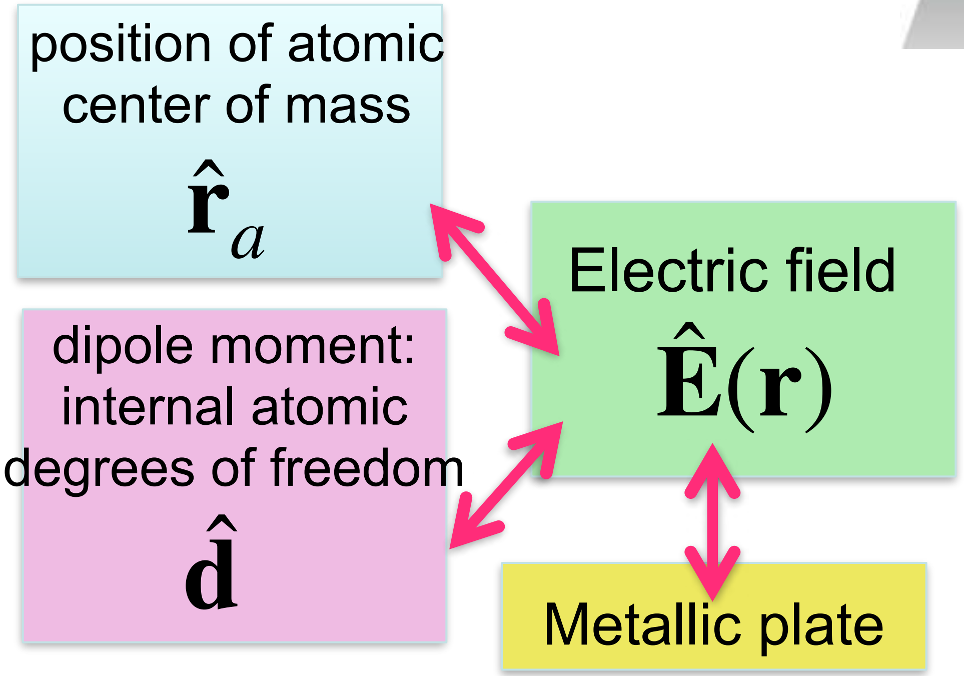
F Impens, C Ccapa-Ttira, R Behunin and PAMN, Phys Rev A  2014

# Atom interferometers as open quantum systems

## Full quantum theory of Casimir interferometers

Atomic center-of-mass as an **open quantum system :** coupling with electromagnetic field and atomic dipole





position of atomic center of mass

$$\hat{\mathbf{r}}_a$$

dipole moment: internal atomic degrees of freedom

$$\hat{\mathbf{d}}$$

Electric field

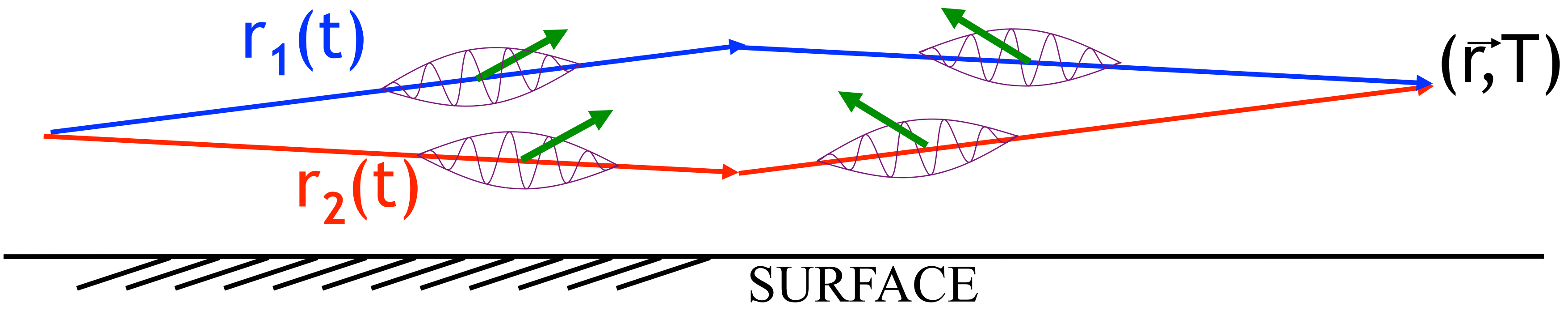
$$\hat{\mathbf{E}}(\mathbf{r})$$

Metallic plate

Hamiltonian in the electric dipole approx.

$$\hat{H}_{\mathrm{AF}} = -\hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\hat{\mathbf{r}}_a)$$

# Casimir disturbance of the environment by the system



Initial state: $|\psi(0)\rangle = \frac{1}{\sqrt{2}}\underbrace{\left(|\psi_E^1(0)\rangle + |\psi_E^2(0)\rangle\right)}_{\text{external/CM}} \otimes \underbrace{|\Psi_{DF}(0)\rangle}_{\text{internal dipole + field}}$

Interaction Hamiltonian: $\hat{H}_{\mathrm{AF}} = -\hat{\mathbf{d}}\cdot\hat{\mathbf{E}}(\hat{\mathbf{r}}_a)$

Final entangled state: $|\psi(T)\rangle = \frac{1}{\sqrt{2}}|\psi_E^1(T)\rangle\otimes|\Psi_{DF}^1(T)\rangle + \frac{1}{\sqrt{2}}|\psi_E^2(T)\rangle\otimes|\Psi_{DF}^2(T)\rangle$

$$|\Psi_{DF}^k(T)\rangle = \mathcal{T}e^{-\frac{i}{\hbar}\int_0^T dt\,\hat{\tilde{H}}_{AF}(r_k(t))}|\Psi_{DF}(0)\rangle$$

Coherence: $\rho_{12}(\mathbf{r},\mathbf{r}';T) = \frac{1}{2}\langle\mathbf{r}|\psi_E^1(T)\rangle\boxed{\langle\Psi_{DF}^2(T)|\Psi_{DF}^1(T)\rangle}\langle\psi_E^2(T)|\mathbf{r}'\rangle$

↘ $\equiv e^{i\Phi_{12}}$

***Influence of the Environment!***

# Atom interferometers as open quantum systems

Interaction Hamiltonian: $\hat{\tilde{H}}_{AF}(\mathbf{r}_k(t),t) = -\hat{\mathbf{d}}(t) \cdot \hat{\mathbf{E}}(\mathbf{r}_k(t),t)$

Effect of the environment

Influence functional: $e^{i\Phi_{12}} = \langle \psi_{\mathrm{DF}}^{(2)}(T) \,|\, \psi_{\mathrm{DF}}^{(1)}(T) \rangle$

$$e^{i\Phi_{12}} = \langle \Psi_{DF}(0) | \widetilde{\mathcal{T}} e^{\frac{i}{\hbar}\int_0^T dt \hat{\tilde{H}}_{AF}(r_2(t))} \mathcal{T} e^{-\frac{i}{\hbar}\int_0^T dt \hat{\tilde{H}}_{AF}(r_1(t))} | \Psi_{DF}(0) \rangle$$

Imaginary part of $\Phi_{12}$ : decoherence

Real part of $\Phi_{12}$: local and **non-local** interferometric phases

# Atom interferometers as open quantum systems

## Second-order term obtained from first-order along each path

Influence functional:

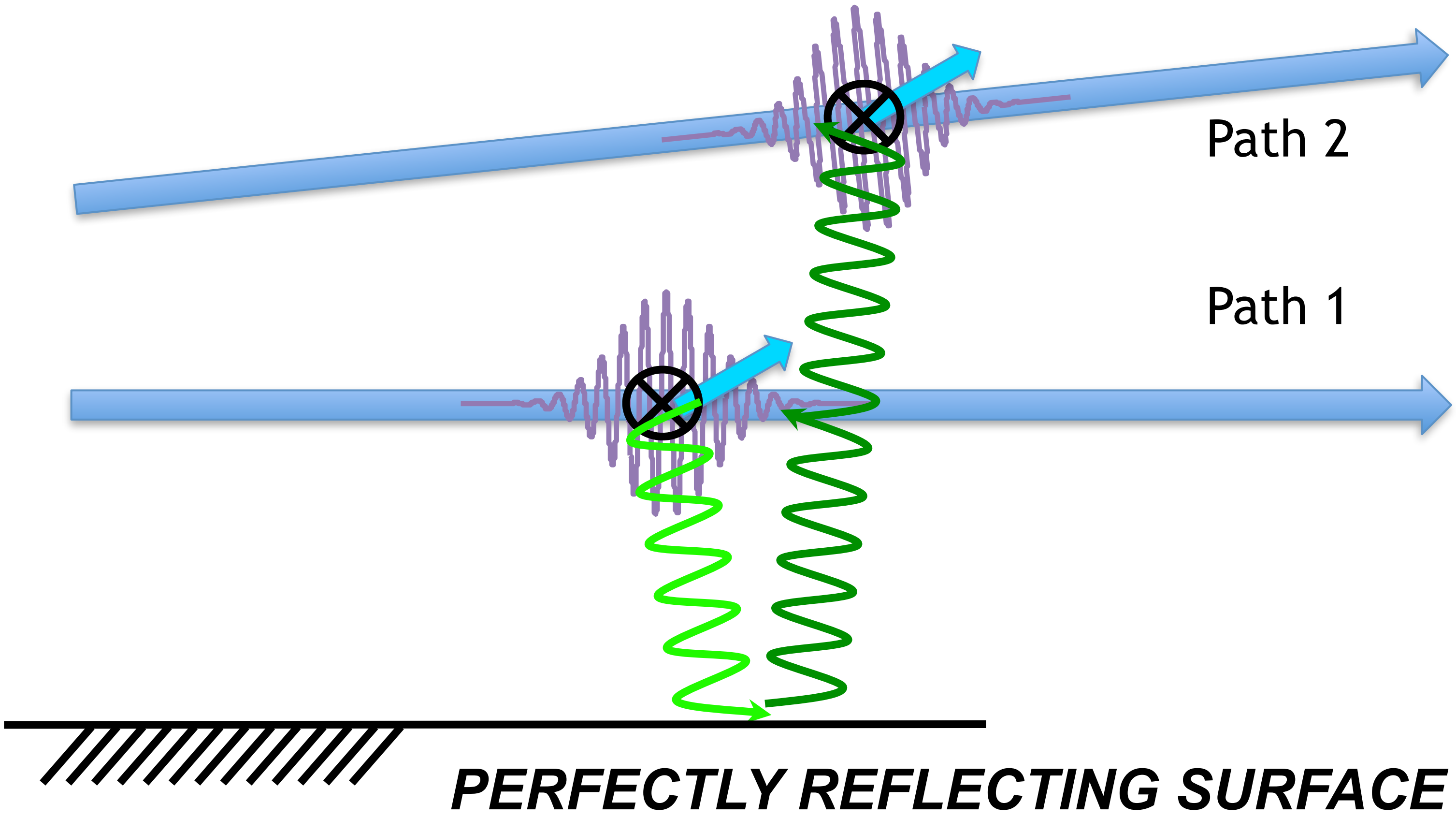
$$e^{i\Phi_{12}} = \langle \Psi_{DF}(0)| \overbrace{\tilde{\mathcal{T}} e^{\frac{i}{\hbar}\int_0^T dt\, \hat{\tilde{H}}_{AF}(r_2(t))}}^{\boxed{2}} \overbrace{\mathcal{T} e^{-\frac{i}{\hbar}\int_0^T dt\, \hat{\tilde{H}}_{AF}(r_1(t))}}^{\boxed{1}} |\Psi_{DF}(0)\rangle$$

***NON-LOCAL DOUBLE-PATH DIAGRAM!***

# Casimir Interactions: Diagrammatic Picture

Influence of the environment:

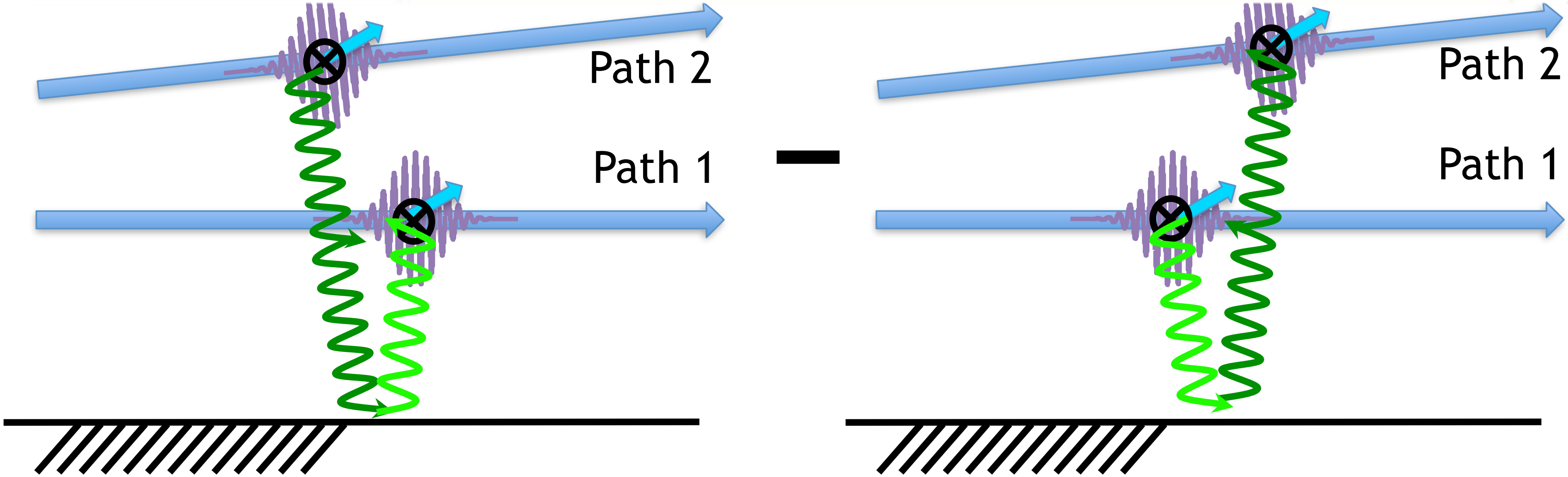
$$e^{i\Phi_{12}} = \langle\Psi_{DF}(0)|\tilde{\mathcal{T}} e^{\frac{i}{\hbar}\int_0^T dt\,\hat{\tilde{H}}_{AF}(r_2(t))}\mathcal{T} e^{-\frac{i}{\hbar}\int_0^T dt\,\hat{\tilde{H}}_{AF}(r_1(t))}|\Psi_{DF}(0)\rangle$$



# Casimir Interactions: Diagrammatic Picture

Non-local double path atomic phase:

$$\phi_{12}^{\mathrm{DP}} = \frac{1}{4}\iint_0^T dt'dt \Big[ g_{\hat{d}}^{H}(t,t')\left(\mathcal{G}_{\hat{\mathbf{E}}}^{R,S}(r_1(t),r_2(t')) - \mathcal{G}_{\hat{\mathbf{E}}}^{R,S}(r_2(t),r_1(t'))\right)$$
$$+ g_{\hat{d}}^{R}(t,t')\left(\mathcal{G}_{\hat{\mathbf{E}}}^{H,S}(r_1(t),r_2(t')) - \mathcal{G}_{\hat{\mathbf{E}}}^{H,S}(r_2(t),r_1(t'))\right)\Big]$$



Fluctuations:

$$G^{H}_{\hat{\mathbf{O}},\,ij}(x;x') = \frac{1}{\hbar}\langle\{\hat{O}_i^f(x),\hat{O}_j^f(x')\}\rangle$$

Linear response susceptibilities:

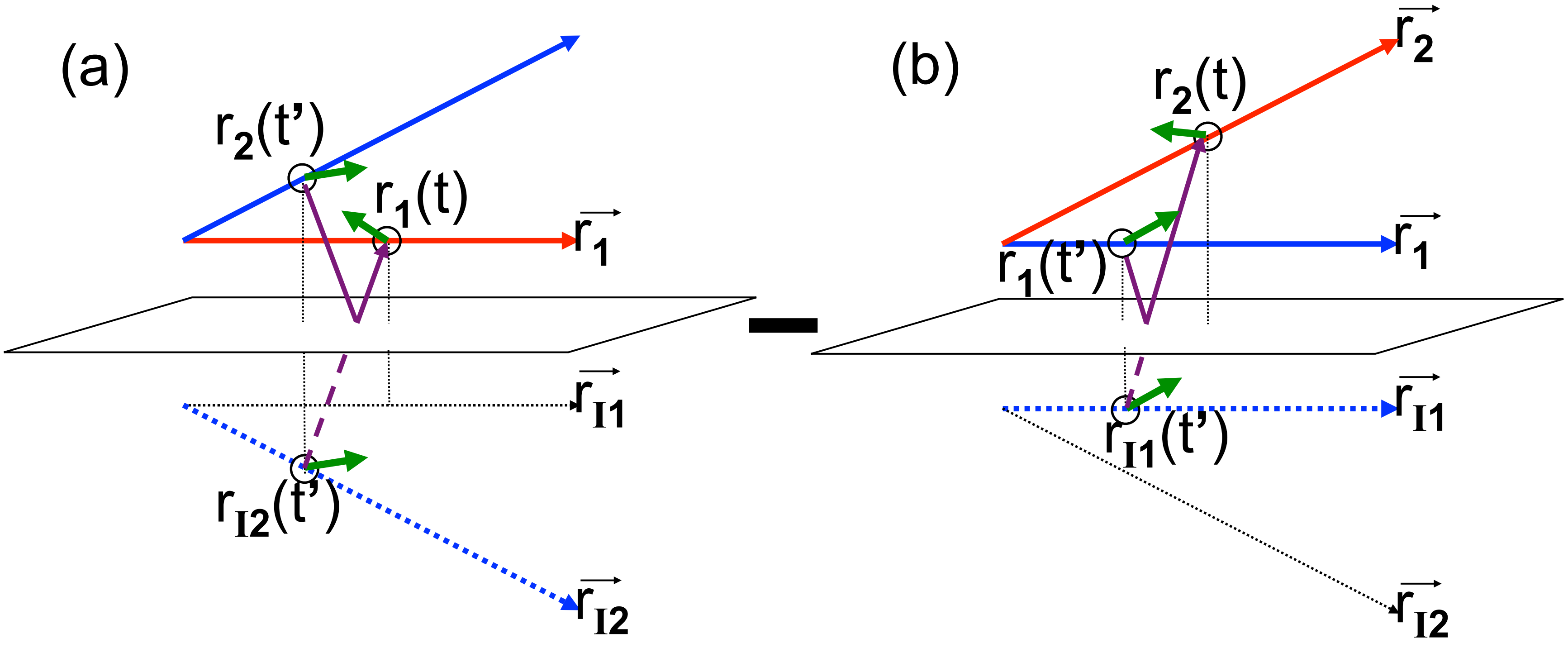
$$G^{R}_{\hat{\mathbf{O}},\,ij}(t,t') = \frac{i}{\hbar}\theta(t-t')\langle[\hat{O}_i^f(t),\hat{O}_j^f(t')]\rangle$$

# **Dynamical** Casimir-like non-local atomic phase

**Atom Interferometer:**

One arm parallel to the plate

Other arm going away from the plate

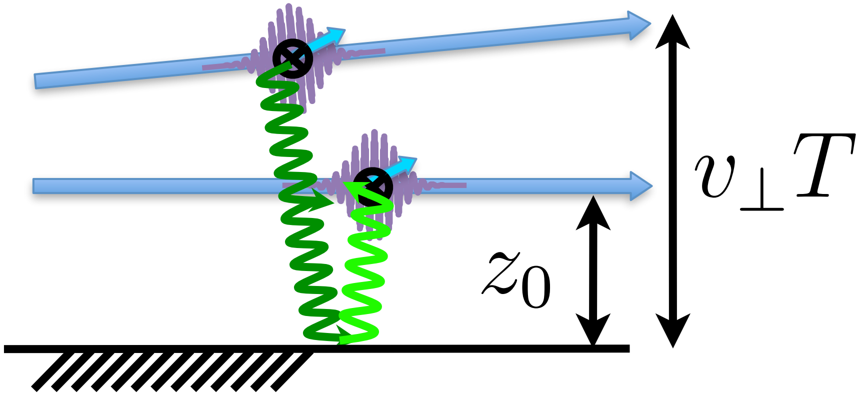


***«Cross-talks» between the two paths***

***Asymmetry avoids cancellation!***

# Non-local double-path Casimir atomic phase

**Double-path phase:**



$$\phi_{12}^{\mathrm{DP}} = \frac{3\pi}{4\lambda_0}\left(\frac{\alpha(0)}{4\pi\epsilon_0}\right)\frac{1}{z_0^2}$$

For narrow wave-packets and in the saturation regime where $z_0 \ll v_\perp T \ll \lambda_0$

**$^{87}$Rb atom:**

$\alpha_{\mathrm{Rb}}(0)/(4\pi\epsilon_0) = 4.72\times10^{-29}\mathrm{m}^3$ Atomic polarizability

$5s_{1/2}-5p_{1/2}$ and $5s_{1/2}-5p_{3/2}$ transitions

Distance of the wave-packet center to the plate: $z_0 = 20\ \mathrm{nm}$

**Narrow atomic packets:** $\phi_{12}^{\mathrm{narrow,DP}} = 3\times10^{-7}\ \mathrm{rad}$

**Wide atomic packets:** $\phi_{12}^{\mathrm{wide,DP}} = 3\times10^{-6}\ \mathrm{rad}$

# Quantum Sagnac effect

# GHz rotation of optically trapped nanoparticles

nature nanotechnology

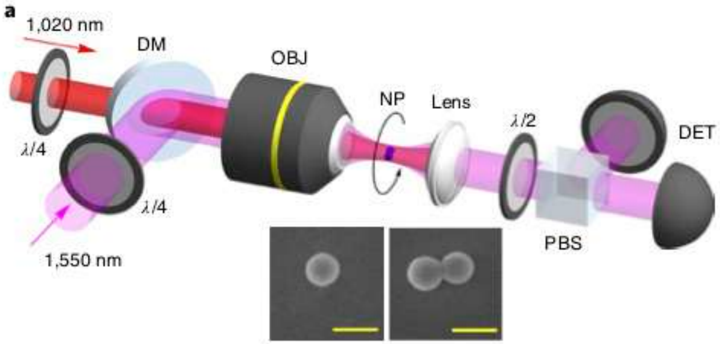
LETTERS

https://doi.org/10.1038/s41565-019-0605-9

**Ultrasensitive torque detection with an optically levitated nanorotor**

Jonghoon Ahn[1], Zhujing Xu[2], Jaehoon Bang[1], Peng Ju[2], Xingyu Gao[2] and Tongcang Li[1,2,3,4*]

**vacuum. Our system does not require complex nanofabrication. Moreover, we drive a nanoparticle to rotate at a record high speed beyond 5 GHz (300 billion r.p.m.). Our calculations**



Featured in Physics

GHz Rotation of an Optically Trapped Nanoparticle in Vacuum

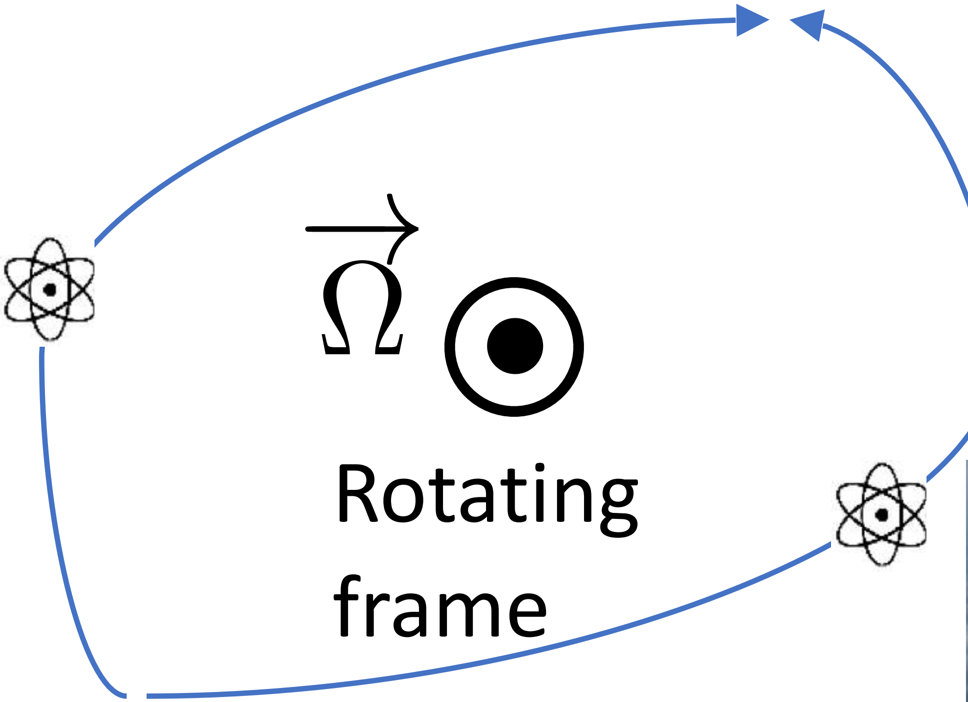
René Reimann, Michael Doderer, Erik Hebestreit, Rozenn Diehl, Martin Frimmer, Dominik Windey, Felix Tebbenjohanns, and Lukas Novotny

Phys. Rev. Lett. **121**, 033602 – Published 20 July 2018; Erratum Phys. Rev. Lett. **126**, 159901 (2021)

Physics See Focus story: The Fastest Spinners

Opportunity to probe dynamical Casimir effects….?

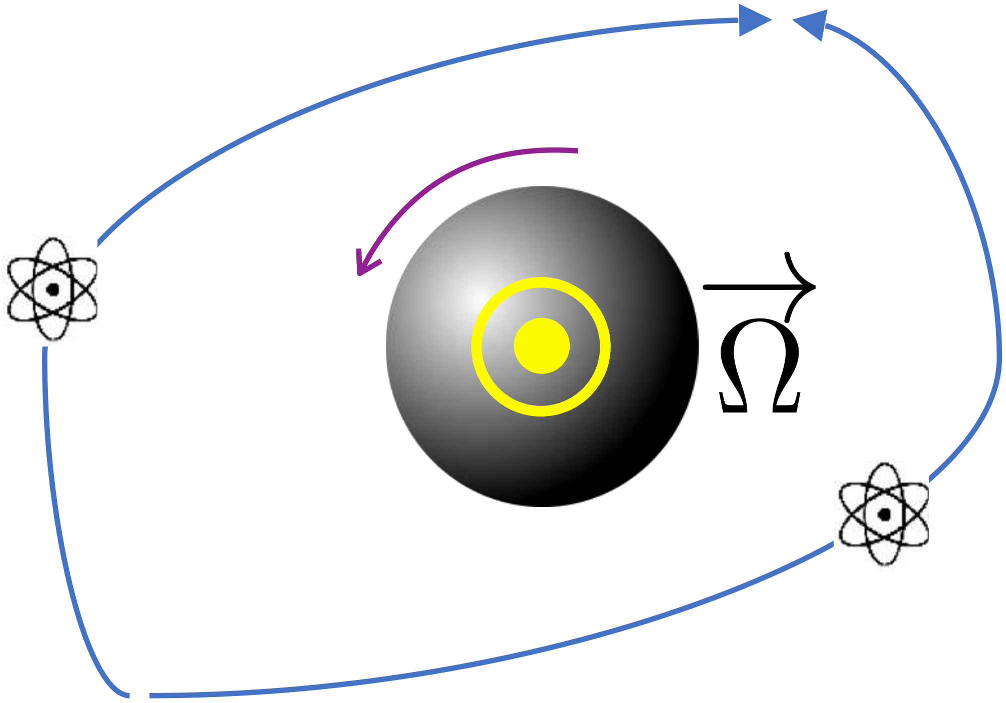
# Sagnac Atom Interferometer



Rotating frame

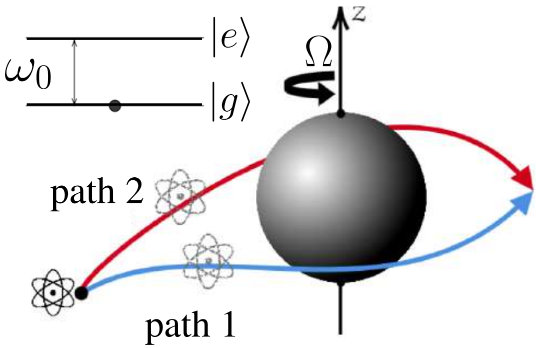
Ex: embarked atom interferometer

# Sagnac effect in an inertial frame?



Inertial frame and rotating conductor

# Quantum Sagnac phase near a spinning particle



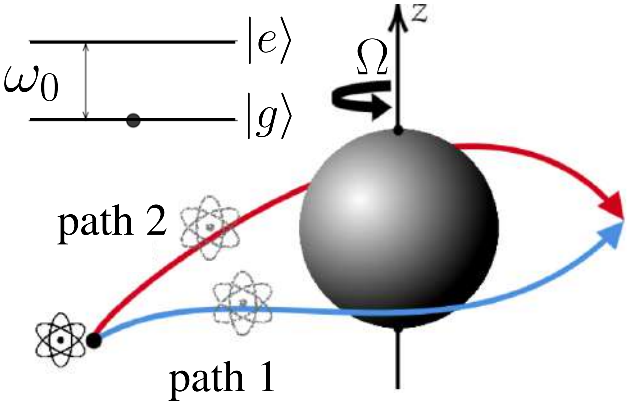
Casimir phase:

$$\Delta\phi_{12} = \varphi_{11} - \varphi_{22} + \varphi_{12} - \varphi_{21}$$

**Spinning nano-particle**

$$\varphi_{kl} = \frac{1}{4} \iint_{-\frac{T}{2}}^{\frac{T}{2}} dt\, dt' \left[ g^{H}_{\hat{\mathbf{d}}}(t,t') \mathcal{G}^{R,S}_{\hat{\mathbf{E}}}(\mathbf{r}_k(t), t; \mathbf{r}_l(t'), t') + (R \leftrightarrow H) \right]$$

# Quantum Sagnac phase



Local Quantum Sagnac phase (in the non-retarded approximation)

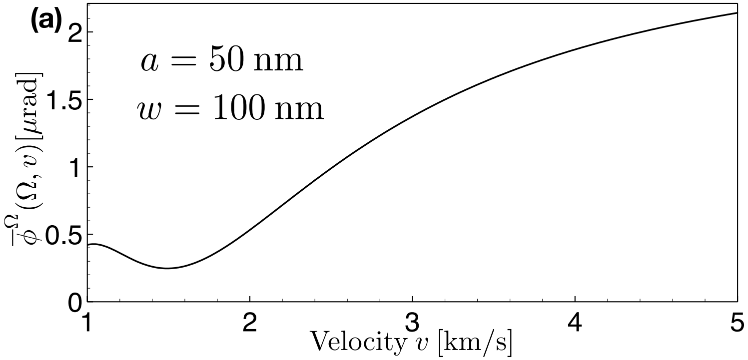
$$\phi^{\Omega}_{\mathrm{vdW,k}} = \frac{9}{2}\,\frac{\omega_0\alpha_0^{\mathrm{A}}\tilde{\alpha}''_{S,R}(\omega_0)}{(4\pi\epsilon_0)^2}\int_{\mathcal{P}_k} d\mathbf{r}\cdot\frac{\mathbf{\Omega}\times\mathbf{r}}{r^8}$$

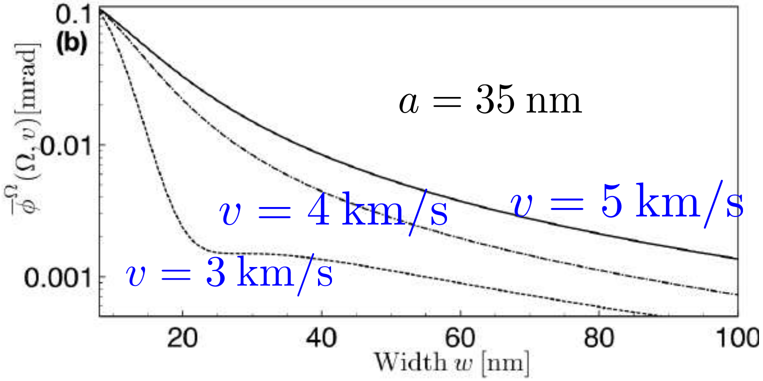
Real part of the spherical particle polarizability $\tilde{\alpha}_{S,R}(\omega) = \mathrm{Re}[\alpha_S(\omega)]$

$\alpha_0^A$ = static atomic polarizability

G. C. Matos, Reinaldo de Melo e Souza, PAMN, and F Impens, Phys. Rev. Lett. **127**, 270401 (2021).

# Estimation of the Quantum Sagnac phase in an atom-Interferometer





Atomic wave-packets of finite width

Total phase = quasi-static van der Waals
+ quantum Sagnac phase

$$\phi(\Omega, x, z, v) = \phi^{\mathrm{vdW}}(x, z, v) + \phi^{\Omega}(x, z)$$

Accessible quantum Sagnac phase

$$\overline{\phi}^{\Omega}(\Omega, v) \equiv \overline{\phi}(\Omega, v) - \overline{\phi}(0, v)$$

averaging over wave-packet width
(as in Alexander D. Cronin and John D. Perreault, Phys. Rev. A 70, 043607 (2004))

$\Omega = 2\pi \times 5\,\mathrm{GHz}$

Nanosphere radius $a = 30 - 50\,\mathrm{nm}$

Na atoms

Atomic beam of width $w = 10 - 100\,\mathrm{nm}$

Atomic velocities $v = 1 - 5\,\mathrm{km/s}$

$$\overline{\phi}^{\Omega}(\Omega, v) \simeq 0.1\,\mathrm{mrad}$$

# Conclusion:

- Influence of the enviroment of the system of interest: decoherence, phase shift in an atom interferometer

- Dynamical Casimir effects: emission of photons, non-unitary non-local phase in an atom interferometer; quantum Sagnac phase

- Methods: master equation/Fokker-Planck equation; influence functional

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- INCT/FAPESP - Complex Fluids
- CNPq, CAPES
- KITP - UCSB

Thank you!