

Quantum decoherence, Casimir effect, quantum vacuum fluctuations

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Current team

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Previous collaborations

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2022 †

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Carlos Farina

Los Alamos Nat Lab

Diego Dalvit

and also several discussions with Ricardo Decca (IUPUI), Kanu Sinha (Arizona State and University of Arizona) and Clemens Jakubec (graduate student at U. Vienna)

Lecture 2

Part A) Decoherence of massive particles by radiation pressure - continuation

- The quantum vacuum and the Casimir effect
- Dynamical Casimir effect
- Decoherence by radiation pressure: master equation, results at zero and finite temperatures

The quantum vacuum and the Casimir effect

The quantum vacuum

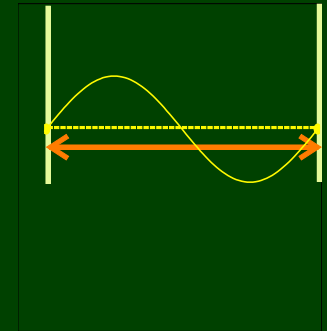
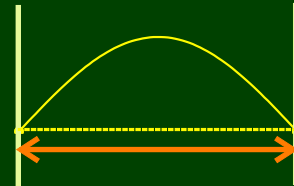
Quantum electromagnetic field

Ground state (no photon): quantum vacuum

E_{vac} = energy corresponding to quantum zero-point fluctuations of the electromagnetic field

sum over normal
modes α

$$E_{\text{vac}} = \sum_{\alpha} \frac{\hbar \omega_{\alpha}}{2}$$



Problem: E_{vac} is always infinite!

The quantum vacuum

Does the vacuum energy have a physical meaning ?

W. Pauli (1933) : “..here it is more consistent, in contrast with the material oscillator, to **not introduce a zero-point energy** ($\hbar\omega/2$) per degree of freedom. On one hand, this energy would lead to an **infinitely large** energy density, and on the other hand it would not be observable since it cannot be emitted, absorbed, or scattered, and hence cannot be contained between walls and, as evident from common experience, **does not produce any gravitational effect.**”



W. Pauli

1948 – Casimir finds that he could explain the interaction potential between ground state atoms in terms of variations of the vacuum energy with respect to the atom-atom distance

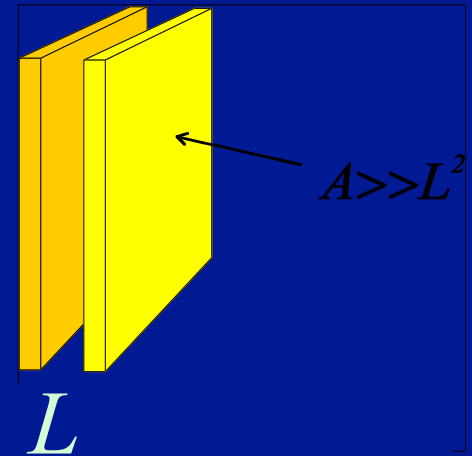


Hendrik BG Casimir

The quantum vacuum

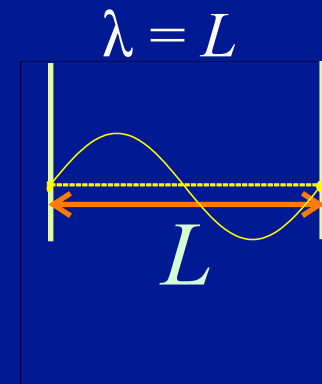
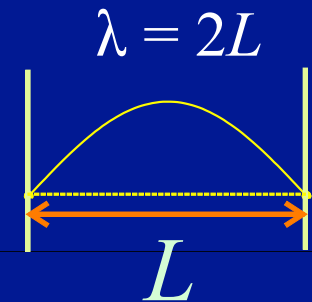
Simplest example: two neutral metallic plates in vacuum - distance L .
Electromagnetic normal modes: frequencies depend on separation L !

$$E_{\text{vac}} = \sum_{\alpha} \frac{\hbar \omega_{\alpha}}{2}$$



Smallest wavelength that fits between plates: $2L$

Modes with $\lambda > 2L$ (frequencies $\omega < \pi c/L$) are suppressed!



Casimir effect

Vacuum energy depends on the cavity length L

→ (usually) attractive force between neutral plates

Isolating the joint effect of the two plates: Casimir energy

$$E_C = -\frac{\pi^2 \hbar c}{720 L^3} A$$

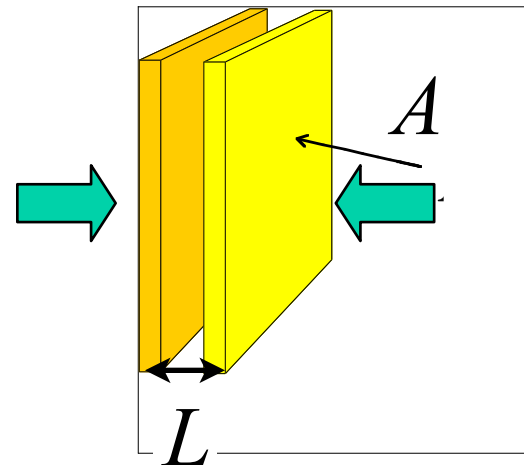
Casimir force between two perfect plane metallic plates at zero temperature

$$F_C = -\frac{dE_C}{dL} = \frac{\pi^2 \hbar c}{240 L^4} A$$

$$F_C/A \approx 12 \text{ N/m}^2 \text{ at } L = 100 \text{ nm}$$

no field-matter coupling constant ...???

connection with van der Waals attraction?



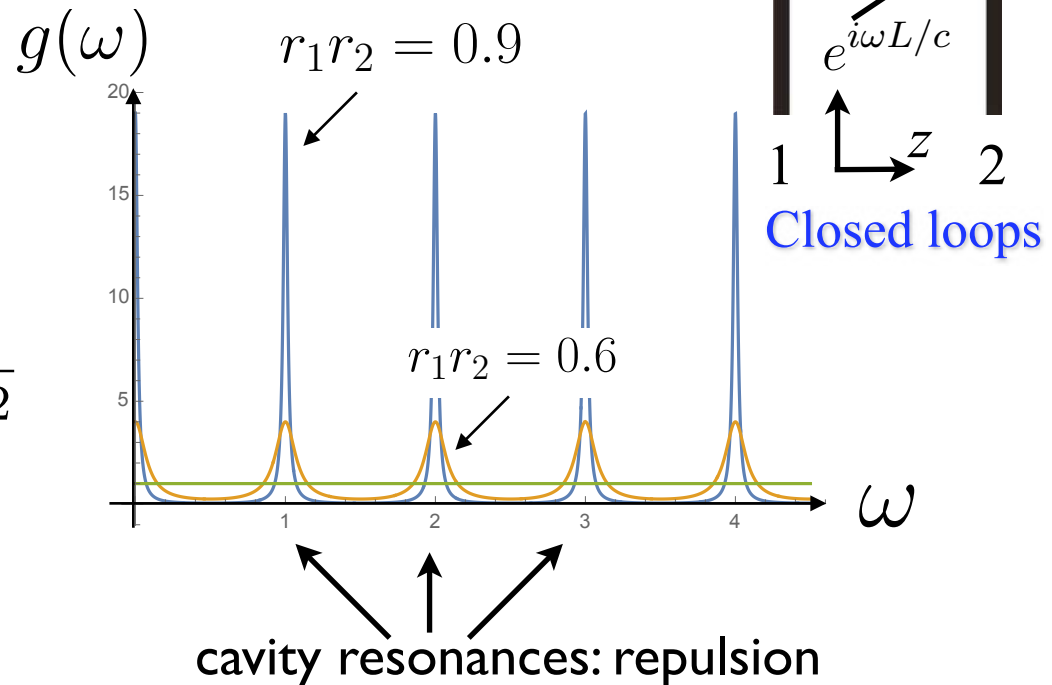
Introduction: Casimir effect

Point-of-view of Quantum Optics/cavity QED:

- density of modes $g(\omega)$ modified by the mirrors
- reflection coefficients as seen by the intracavity field
- example: 1D model

$$F = \int_0^\infty \frac{d\omega}{2\pi} \frac{\hbar\omega}{c} (g(\omega) - 1)$$

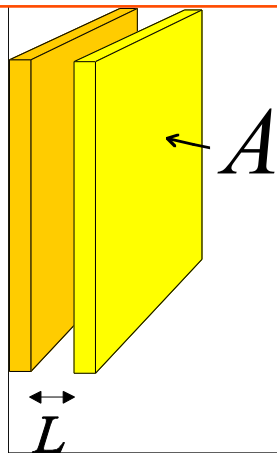
$$g(\omega) = \frac{1 - |r_1 r_2|^2}{|1 - r_1 r_2 e^{2i\omega L/c}|^2}$$



Net force: **attractive** ! (van der Waals ok)

Casimir effect

two metallic mirrors described by
the **plasma model**



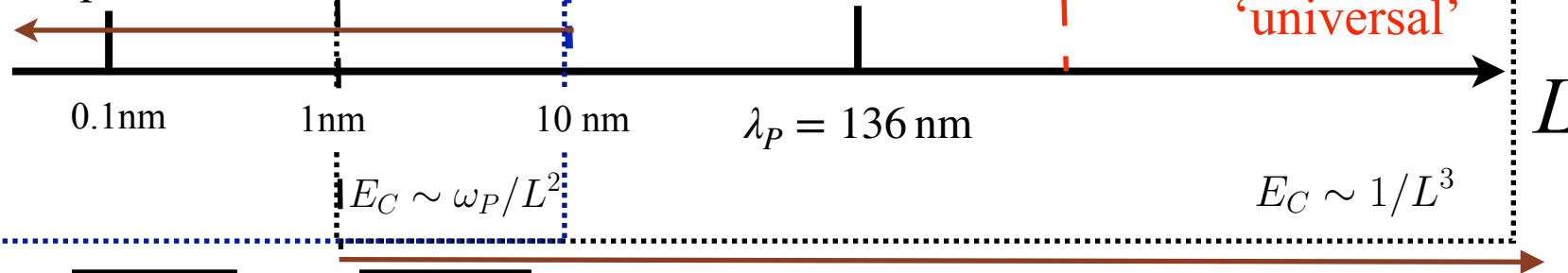
$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

plasma wavelength

$$\lambda_P = \frac{2\pi c}{\omega_P}$$

van der Waals

electron
wavefunctions
overlap



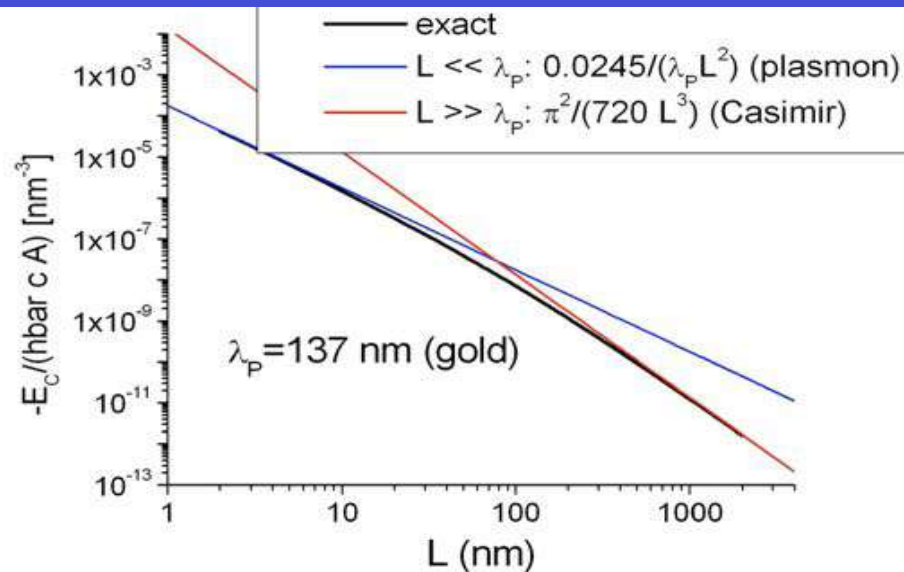
STM

AFM

description in terms of
macroscopic Maxwell
equations



Lifshitz formula



The dynamical Casimir effect

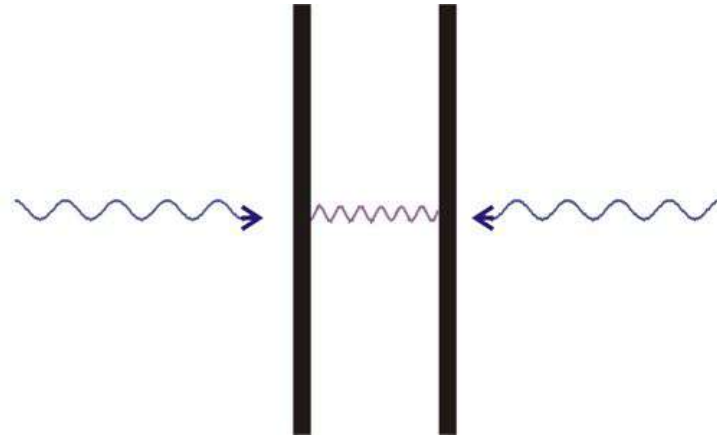
From the Static to the Dynamical Casimir Effect

Casimir force: quantum average of the Maxwell stress tensor taking the field vacuum state ($T = 0$)

2 plates: non-zero average

ex: perfect reflectors -
Casimir 1948

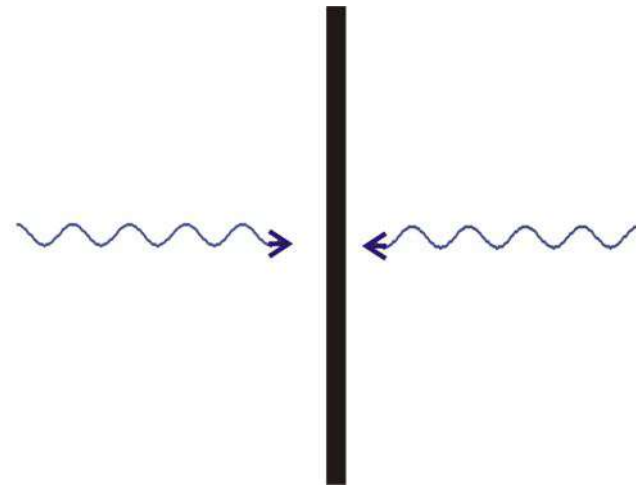
$$\langle F \rangle = F_{\text{PP}} = -\frac{\pi^2 \hbar c A}{240 d^4}$$



single plate in vacuum:

symmetry 

$$\langle F \rangle = 0$$



Dynamical Casimir diffusion

single plate: $\langle \text{vac} | F | \text{vac} \rangle = 0$

..but force fluctuations are present

Barton 1991

Jaekel & Reynaud 1991

Eberlein 1992

....

Dean, Parsegian & Podgornik 2013

Any particle scattering vacuum fluctuations/
thermal photons will develop (quantum)

Brownian fluctuations

open quantum system: environment = quantum
electromagnetic field

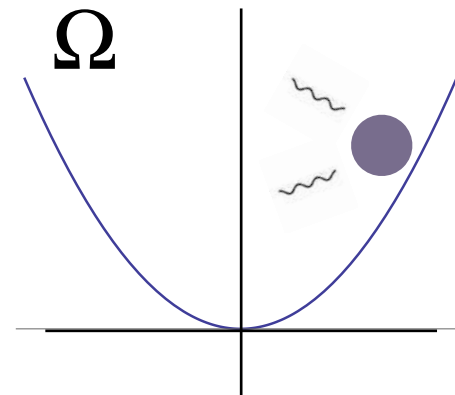
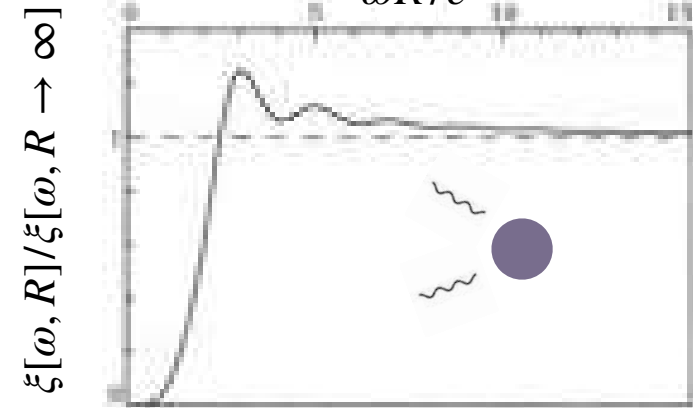
at zero temperature, decoherence arises from the
emission of photon pairs out of the vacuum state

2nd example: sphere radius R
(PAMN & S Reynaud 1993)

$$\xi(t) \equiv \langle \text{vac} | [F(t), F(0)] | \text{vac} \rangle \quad \text{Freq. domain: } \tilde{\xi}[\omega]$$

Large sphere:

$$\omega R/c \gg 1 \implies \tilde{\xi}[\omega] \approx \frac{2\hbar^2 \omega^5 R^2}{45\pi c^4} \frac{1}{\omega R/c}$$



Dynamical Casimir diffusion

open quantum system: environment = quantum electromagnetic field

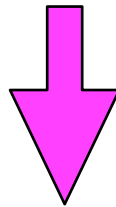
at zero temperature, decoherence arises from the emission of photon pairs out of the vacuum state

Example: initial quantum superposition of ground and coherent states

$$|\Psi\rangle_0 = \frac{1}{\sqrt{2}} (|0\rangle_{\text{CM}} + |\alpha_0\rangle_{\text{CM}}) \otimes |\text{vac}\rangle_{\text{F}}$$

evolution

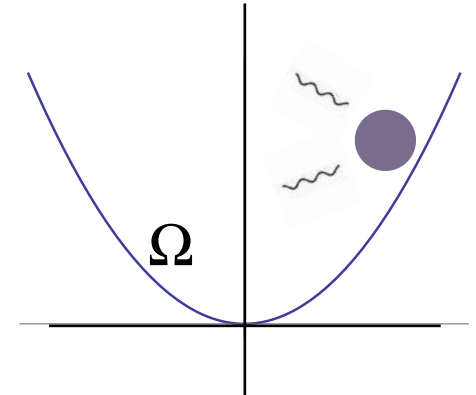
$$|\Psi\rangle_t = e^{-iHt/\hbar} |\Psi\rangle_0$$



$$|\Psi\rangle_t = \frac{1}{\sqrt{2}} \left[|0\rangle_{\text{CM}} \otimes |\text{vac}\rangle_{\text{F}} + |\alpha(t)\rangle_{\text{CM}} \otimes \left(\underline{B(t)} |\text{vac}\rangle_{\text{F}} + \int d\omega_1 d\omega_2 \underline{b(\omega_1, \omega_2, t)} |1_{\omega_1}, 1_{\omega_2}\rangle_{\text{F}} \right) \right]$$

amplitude of persistence
in the vacuum state
amplitude of emission
of pair of photons

$$\alpha(t) = \alpha_0 e^{-i\Omega t}$$



Emission of dynamical Casimir photons provide ‘which-way’ information about the quantum alternatives or ‘paths’ [D. A. R. Dalvit & PAMN 2000](#)

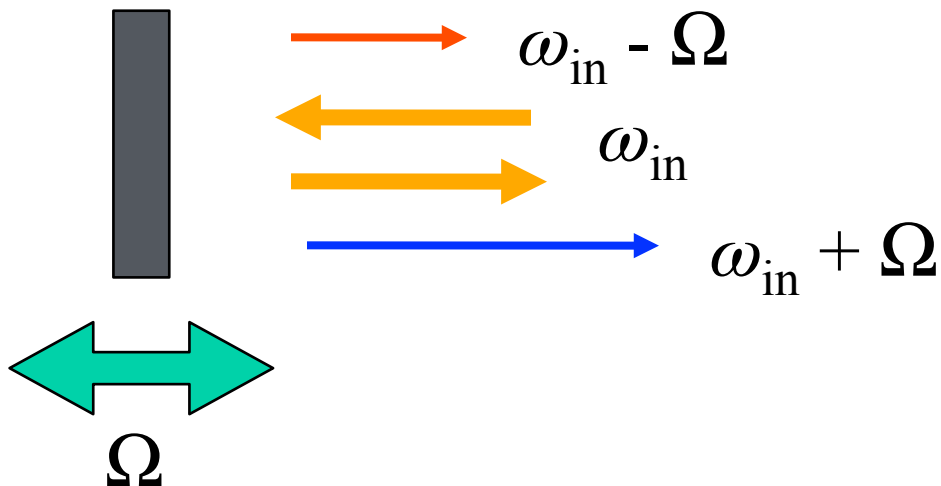
Dynamical Casimir effect

Understanding the dynamical Casimir effect

Take a mirror oscillating at frequency Ω

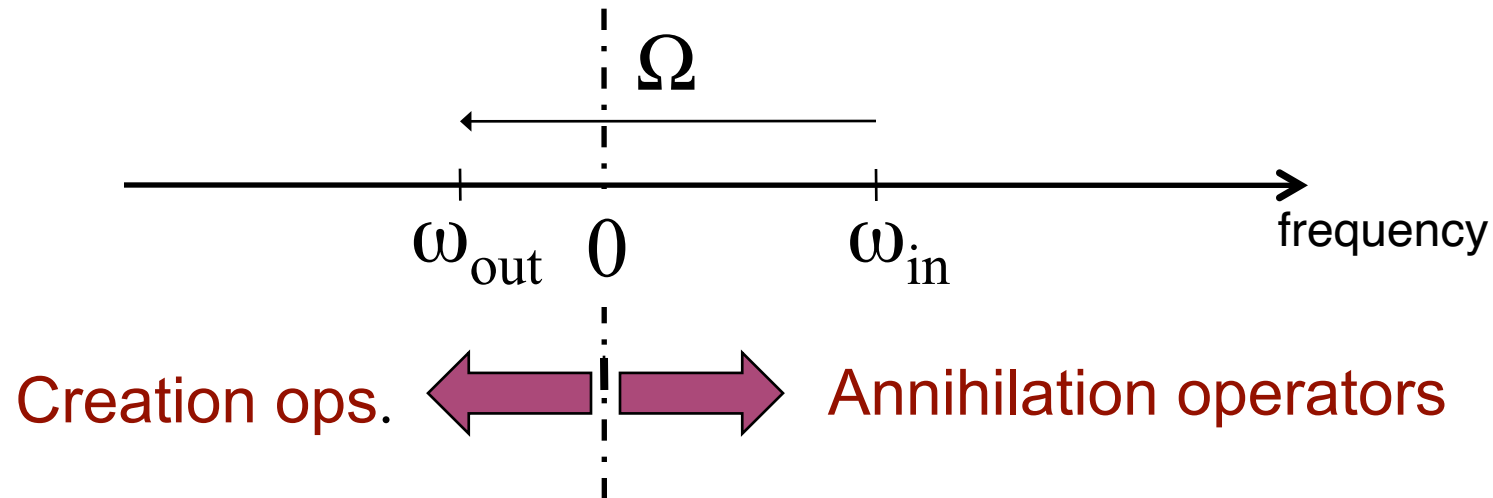
Reflection of field of frequency ω_{in} generates frequency sidebands

$$\omega_{\text{in}} + \Omega, \omega_{\text{in}} - \Omega$$



Dynamical Casimir effect

Non-adiabatic mix up of positive and negative field frequencies



Bogoliubov transformation between input and output ops.:

$$a_{\text{out}} = \alpha a_{\text{in}} + \beta a_{\text{in}}^{\dagger}$$

Average number of dynamical Casimir photons

$$\langle 0_{\text{in}} | a_{\text{out}}^{\dagger} a_{\text{out}} | 0_{\text{in}} \rangle = \beta^2 \langle 0_{\text{in}} | a_{\text{in}} a_{\text{in}}^{\dagger} | 0_{\text{in}} \rangle = \beta^2$$

Dynamical Casimir effect

Important property:

No radiation in the quasi-static limit

Radiation is a consequence of a sudden change of the boundary conditions

Mechanical frequency Ω
induces the generation of photons at frequencies
 $\omega \leq \Omega$
in the non-relativistic regime

Dynamical Casimir effect

single, non-relativistic plane mirror in 3D,
harmonic oscillation frequency Ω

Spectrum displays symmetry around $\Omega/2$:
pair of photons with the same polarization
and frequencies ω_1, ω_2 such that

$$\omega_1 + \omega_2 = \Omega$$

More TM than TE photons:

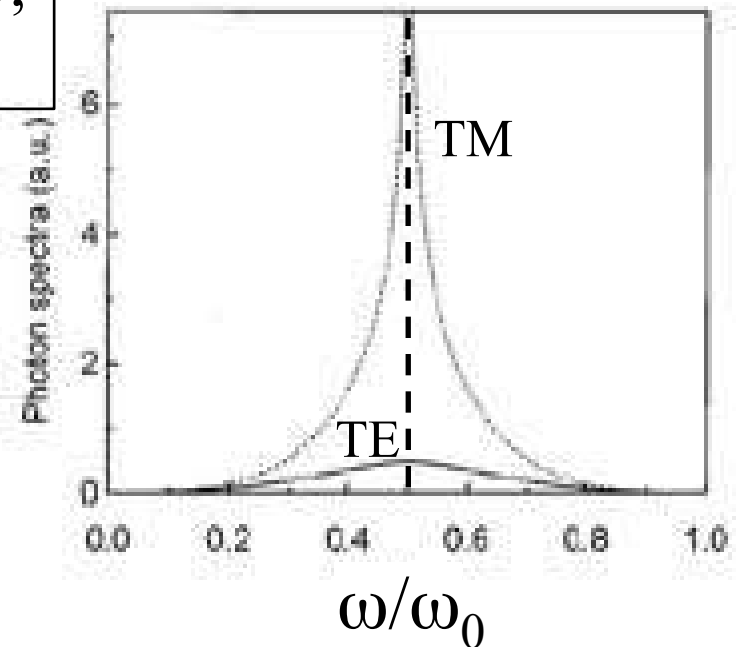
$$\frac{dN_{\text{TM}}}{dt} = 11 \frac{dN_{\text{TE}}}{dt}$$

Total photon emission rate



$$v_{\text{max}} = \Omega \times \text{amplitude}$$

$$\lambda_{\text{mech}} = \frac{2\pi c}{\Omega}$$



$$\frac{dN}{dt} = \frac{1}{15} \frac{A}{\lambda_{\text{mech}}^2} \left(\frac{v_{\text{max}}}{c} \right)^2 \Omega$$