Curso Mecânica Clássica I - 2020/1

Lista 1a

1. Demonstre a identidade de Jacobi

$$(\mathbf{A}\times\mathbf{B})\times\mathbf{C}+(\mathbf{C}\times\mathbf{A})\times\mathbf{B}+(\mathbf{B}\times\mathbf{C})\times\mathbf{A}=0$$

2. No curso introduzimos a delta de Kronecker δ_{ij} e o símbolo de Levi-Civita ϵ_{ijk}

$$\delta_{ij}=\begin{cases}1, & i=j\\ 0, & i\neq j\end{cases}\qquad \epsilon_{ijk}=\begin{cases}1, & ijk=[123]\\ -1, & ijk=[213]\\ 0, & \text{resto}\end{cases}\tag{1}$$

   (a) Verifique (e memorize!) a célebre identidade

$$\epsilon_{kij}\epsilon_{klm}=\delta_{il}\delta_{jm}-\delta_{im}\delta_{jl}$$

   (b) A partir dela, demostre

$$(\mathbf{A}\times\mathbf{B})\cdot(\mathbf{C}\times\mathbf{D})=(\mathbf{A}\cdot\mathbf{C})(\mathbf{B}\cdot\mathbf{D})-(\mathbf{A}\cdot\mathbf{D})(\mathbf{B}\cdot\mathbf{C})$$

3. Sabendo que

$$A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\tag{2}$$

demonstre

$$A^2=\begin{bmatrix}\cos 2\theta & -\sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{bmatrix}\qquad A^3=\begin{bmatrix}\cos 3\theta & -\sin 3\theta\\ \sin 3\theta & \cos 3\theta\end{bmatrix}\tag{3}$$

$$(\mathbf{A}\times\mathbf{B})\times\mathbf{C}-\mathbf{A}\times(\mathbf{B}\times\mathbf{C})=\mathbf{B}\times(\mathbf{C}\times\mathbf{A})$$

$$(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}=(\mathbf{C}\times\mathbf{A})\cdot\mathbf{B}=(\mathbf{B}\times\mathbf{C})\cdot\mathbf{A}$$

$$\epsilon_{ijk}A_jB_k\quad C_i$$